AD-A081 317   OAKLAND UNIV ROCHESTER MICH SCHOOL OF ENGINEERING   F/G 20/11

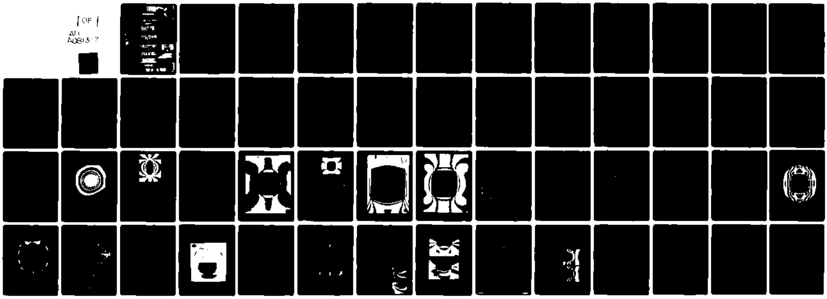
LIGHTER AND STRONGER,(U)

FEB 80   A J DURELLI, K RAJAIAH   N00014-76-C-0487

UNCLASSIFIED   53   NL

1 OF 1

AD A081317



END
DATE
FILMED
4-80
DDC

14 53

6 **LIGHTER AND STRONGER**,

by

10 A. J. Durelli [redacted] K. Rajaiah

Sponsored by

Office of Naval Research
Department of the Navy
Washington, D.C. 20025

on

15 Contract No. N0014-76-C-0487 NSF-ENG77-07174
O.U. Project No. 31313-24
Report No. 53

and

National Science Foundation
Washington, D.C. 20550

on

Grant No. ENG77-07974
O.U. Project No. 32110-18

12 54

School of Engineering
Oakland University
Rochester, Michigan 48063

11 Feb[redacted]80

405252

LIGHTER AND STRONGER

by

A. J. Durelli and K. Rajaiah

ABSTRACT

A new method has been developed that permits the direct design of shapes of two-dimensional structures and structural components, loaded in their plane, within specified design constrains and exhibiting optimum distribution of stresses. The method uses photoelasticity and requires a large field diffused light polariscope. Several problems of optimisation related to the presence of holes in finite and infinite plates, subjected to uniaxial and biaxial loadings, are solved parametrically.

Some unexpected results have been found: 1) the optimum shape of a large hole in a bar of finite width, subjected to uniaxial load, is "quasi" square, but the transverse boundary has the configuration of a "hat"; 2) for the small hole in the large plate, a "barrel" shape has a lower s.c.f. than the circular hole and appreciably higher coefficient of efficiency; 3) the optimum shape of a tube, subjected to diametral compression, has small "hinges" and is much lighter and stronger than the circular tube. Applications are also shown to the design of dove-tails and slots in turbine blades and rotors, and to the design of star-shaped solid propellant grains for rockets.

LIGHTER AND STRONGER

TABLE OF CONTENTS

Previous Technical Reports to the Office of Naval Research

1. A. J. Durelli, "Development of Experimental Stress Analysis Methods to Determine Stresses and Strains in Solid Propellant Grains"--June 1962. Developments in the manufacturing of grain-propellant models are reported. Two methods are given: a) cementing routed layers and b) casting.

2. A. J. Durelli and V. J. Parks, "New Method to Determine Restrained Shrinkage Stresses in Propellant Grain Models"--October 1962. The birefringence exhibited in the curing process of a partially restrained polyurethane rubber is used to determine the stress associated with restrained shrinkage in models of solid propellant grains partially bonded to the case.

3. A. J. Durelli, "Recent Advances in the Application of Photoelasticity in the Missile Industry"--October 1962. Two- and three-dimensional photoelastic analysis of grains loaded by pressure and by temperature are presented. Some applications to the optimization of fillet contours and to the redesign of case joints are also included.

4. A. J. Durelli and V. J. Parks, "Experimental Solution of Some Mixed Boundary Value Problems"--April 1964. Means of applying known displacements and known stresses to the boundaries of models used in experimental stress analysis are given. The application of some of these methods to the analysis of stresses in the field of solid propellant grains is illustrated. The presence of the "pinching effect" is discussed.

5. A. J. Durelli, "Brief Review of the State of the Art and Expected Advance in Experimental Stress and Strain Analysis of Solid Propellant Grains"--April 1964. A brief review is made of the state of the experimental stress and strain analysis of solid propellant grains. A discussion of the prospects for the next fifteen years is added.

6. A. J. Durelli, "Experimental Strain and Stress Analysis of Solid Propellant Rocket Motors"--March 1965. A review is made of the experimental methods used to strain-analyze solid propellant rocket motor shells and grains when subjected to different loading conditions. Methods directed at the determination of strains in actual rockets are included.

7. L. Ferrer, V. J. Parks and A. J. Durelli, "An Experimental Method to Analyze Gravitational Stresses in Two-Dimensional Problems"--October 1965. Photoelasticity and moiré methods are used to solve two-dimensional problems in which gravity-stresses are present.

8. A. J. Durelli, V. J. Parks and C. J. del Rio, "Stresses in a Square Slab Bonded on One Face to a Rigid Plate and Shrunk"--November 1965.
A square epoxy slab was bonded to a rigid plate on one of its faces in the process of curing. In the same process the photoelastic effects associated with a state of restrained shrinkage were "frozen-in." Three-dimensional photoelasticity was used in the analysis.

9. A. J. Durelli, V. J. Parks and C. J. del Rio, "Experimental Determination of Stresses and Displacements in Thick-Wall Cylinders of Complicated Shape"--April 1966.
Photoelasticity and moiré are used to analyze a three-dimensional rocket shape with a star shaped core subjected to internal pressure.

10. V. J. Parks, A. J. Durelli and L. Ferrer, "Gravitational Stresses Determined Using Immersion Techniques"--July 1966.
The methods presented in Technical Report No. 7 above are extended to three-dimensions. Immersion is used to increase response.

11. A. J. Durelli and V. J. Parks. "Experimental Stress Analysis of Loaded Boundaries in Two-Dimensional Second Boundary Value Problems"--February 1967.
The pinching effect that occurs in two-dimensional bonding problems, noted in Reports 2 and 4 above, is analyzed in some detail.

12. A. J. Durelli, V. J. Parks, H. C. Feng and F. Chiang, "Strains and Stresses in Matrices with Inserts,"-- May 1967.
Stresses and strains along the interfaces, and near the fiber ends, for different fiber end configurations, are studied in detail.

13. A. J. Durelli, V. J. Parks and S. Uribe, "Optimization of a Slot End Configuration in a Finite Plate Subjected to Uniformly Distributed Load,"--June 1967.
Two-dimensional photoelasticity was used to study various elliptical ends to a slot, and determine which would give the lowest stress concentration for a load normal to the slot length.

14. A. J. Durelli, V. J. Parks and Han-Chow Lee, "Stresses in a Split Cylinder Bonded to a Case and Subjected to Restrained Shrinkage,"--January 1968.
A three-dimensional photoelastic study that describes a method and shows results for the stresses on the free boundaries and at the bonded interface of a solid propellant rocket.

15. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--August 1968.
This report has been written following a trip conducted by the author through several European countries. A list is given of many of the laboratories doing important experimental stress analysis work and of the people interested in this kind of work. An attempt has been made to abstract the main characteristics of the methods used in some of the countries visited.

16. V. J. Parks, A. J. Durelli and L. Ferrer, "Constant Acceleration Stresses in a Composite Body"--October 1968.
Use of the immersion analogy to determine gravitational stresses in two-dimensional bodies made of materials with different properties.

17. A. J. Durelli, J. A. Clark and A. Kochev, "Experimental Analysis of High Frequency Stress Waves in a Ring"--October 1968.
A method for the complete experimental determination of dynamic stress distributions in a ring is demonstrated. Photoelastic data is supplemented by measurements with a capacitance gage used as a dynamic lateral extensometer.

18. J. A. Clark and A. J. Durelli, "A Modified Method of Holographic Interferometry for Static and Dynamic Photoelasticity"--April 1968.
A simplified absolute retardation approach to photoelastic analysis is described. Dynamic isopachics are presented.

19. J. A. Clark and A. J. Durelli, "Photoelastic Analysis of Flexural Waves in a Bar"--May 1969.
A complete direct, full-field optical determination of dynamic stress distribution is illustrated. The method is applied to the study of flexural waves propagating in a urethane rubber bar. Results are compared with approximate theories of flexural waves.

20. J. A. Clark and A. J. Durelli, "Optical Analysis of Vibrations in Continuous Media"--June 1969.
Optical methods of vibration analysis are described which are independent of assumptions associated with theories of wave propagation. Methods are illustrated with studies of transverse waves in prestressed bars, snap loading of bars and motion of a fluid surrounding a vibrating bar.

21. V. J. Parks, A. J. Durelli, K. Chandrashekhara and T. L. Chen, "Stress Distribution Around a Circular Bar, with Flat and Spherical Ends, Embedded in a Matrix in a Triaxial Stress Field"--July 1969.
A Three-dimensional photoelastic method to determine stresses in composite materials is applied to this basic shape. The analyses of models with different loads are combined to obtain stresses for the triaxial cases.

22. A. J. Durelli, V. J. Parks and L. Ferrer, "Stresses in Solid and Hollow Spheres Subjected to Gravity or to Normal Surface Tractions"--October 1969.
The method described in Report No. 10 above is applied to two specific problems. An approach is suggested to extend the solutions to a class of surface traction problems.

23. J. A. Clark and A. J. Durelli, "Separation of Additive and Subtractive Moiré Patterns"--December 1969.
A spatial filtering technique for adding and subtracting images of several gratings is described and employed to determine the whole field of Cartesian shears and rigid rotations.

24. R. J. Sanford and A. J. Durelli, "Interpretation of Fringes in Stress-Holo-Interferometry"--July 1970.
Errors associated with interpreting stress-holo-interferometry patterns as the superposition of isopachics (with half order fringe shifts) and isochromatics are analyzed theoretically and illustrated with computer generated holographic interference patterns.

25. J. A. Clark, A. J. Durelli and P. A. Laura, "On the Effect of Initial Stress on the Propagation of Flexural Waves in Elastic Rectangular Bars"--December 1970.
Experimental analysis of the propagation of flexural waves in prismatic, elastic bars with and without prestressing. The effects of prestressing by axial tension, axial compression and pure bending are illustrated.

26. A. J. Durelli and J. A. Clark, "Experimental Analysis of Stresses in a Buoy-Cable System Using a Birefringent Fluid"--February 1971.
An extension of the method of photoviscous analysis is presented which permits quantitative studies of strains associated with steady state vibrations of immersed structures. The method is applied in an investigation of one form of behavior of buoy-cable systems loaded by the action of surface waves.

27. A. J. Durelli and T. L. Chen, "Displacements and Finite-Strain Fields in a Sphere Subjected to Large Deformations"--February 1972.
Displacements and strains (ranging from 0.001 to 0.50) are determined in a polyurethane sphere subjected to several levels of diametral compression. A 500 lines-per-inch grating was embedded in a meridian plane of the sphere and moiré effect produced with a non-deformed master. The maximum applied vertical displacement reduced the diameter of the sphere by 27 per cent.

28. A. J. Durelli and S. Machida, "Stresses and Strain in a Disk with Variable Modulus of Elasticity"--March 1972
A transparent material with variable modulus of elasticity has been manufactured that exhibits good photoelastic properties and can also be strain analyzed by moiré. The results obtained suggests that the stress distribution in the disk of variable E is practically the same as the stress distribution in the homogeneous disk. It also indicates that the strain fields in both cases are very different, but that it is possible, approximately, to obtain the stress field from the strain field using the value of E at every point, and Hooke's law.

29. A. J. Durelli and J. Buitrago, "State of Stress and Strain in a Rectangular Belt Pulled Over a Cylindrical Pulley"--June 1972.
Two- and three-dimensional photoelasticity as well as electrical strain gages, dial gages and micrometers are used to determine the stress distribution in a belt-pulley system. Contact and tangential stress for various contact angles and friction coefficients are given.

30. T. L. Chen and A. J. Durelli, "Stress Field in a Sphere Subjected to Large Deformations"--June 1972.
Strain fields obtained in a sphere subjected to large diametral compressions from a previous paper were converted into stress fields using two approaches. First, the concept of strain-energy function for an isotropic elastic body was used. Then the stress field was determined with the Hookean type natural stress-natural strain relation. The results so obtained were also compared.

31. A. J. Durelli, V. J. Parks and H. M. Hasseem, "Helices Under Load"--July 1973.
Previous solutions for the case of close coiled helical springs and for helices made of thin bars are extended. The complete solution is presented in graphs for the use of designers. The theoretical development is correlated with experiments.

32. T. L. Chen and A. J. Durelli, "Displacements and Finite Strain Fields in a Hollow Sphere Subjected to Large Elastic Deformations"--September 1973.
The same methods described in No. 27, were applied to a hollow sphere with an inner diameter one half the outer diameter. The hollow sphere was loaded up to a strain of 30 per cent on the meridian plane and a reduction of the diameter by 20 per cent.

33. A. J. Durelli, H. H. Hasseem and V. J. Parks, "New Experimental Method in Three-Dimensional Elastostatics"--December 1973.
A new material is reported which is unique among three-dimensional stress-freezing materials, in that, in its heated (or rubbery) state it has a Poisson's ratio which is appreciably lower than 0.5. For a loaded model, made of this material, the unique property allows the direct determination of stresses from strain measurements taken at interior points in the model.

34. J. Wolak and V. J. Parks, "Evaluation of Large Strains in Industrial Applications"--April 1974.
It was shown that Mohr's circle permits the transformation of strain from one axis of reference to another, irrespective of the magnitude of the strain, and leads to the evaluation of the principal strain components from the measurement of direct strain in three directions.

35. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--April 1975.
Continuation of Report No. 15 after a visit to Belgium, Holland, Germany, France, Turkey, England and Scotland.

36. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Linear and Non-linear Elastic and Plastic Strains in a Plate with a Big Hole Loaded Axially in its Plane"--July 1975.
Strain analysis of the ligament of a plate with a big hole indicates that both geometric and material non-linearity may take place. The strain concentration factor was found to vary from 1 to 2 depending on the level of deformation.

37. A. J. Durelli, V. Pavlin, J. O. Bühler-Vidal and G. Ome, "Elastostatics of a Cubic Box Subjected to Concentrated Loads"--August 1975.
Analysis of experimental strain, stress and deflection of a cubic box subjected to concentrated loads applied at the center of two opposite faces. The ratio between the inside span and the wall thickness was varied between approximately 5 and 121.

38. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Elastostatics of Cubic Boxes Subjected to Pressure"--March 1976.
Experimental analysis of strain, stress and deflections in a cubic box subjected to either internal or external pressure. Inside span-to-wall thickness ratio varied from 5 to 14.

39. Y. Y. Hung, J. D. Hovanesian and A. J. Durelli, "New Optical Method to Determine Vibration-Induced Strains with Variable Sensitivity After Recording"--November 1976.
A steady state vibrating object is illuminated with coherent light and its image slightly misfocused. The resulting specklegram is "time-integrated" as when Fourier filtered gives derivatives of the vibrational amplitude.

40. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Cyclic Stress Studies by Time-Averaged Photoelasticity"--November 1976.
"Time-averaged isochromatics" are formed when the photographic film is exposed for more than one period. Fringes represent amplitudes of the oscillating stress according to the zeroth order Bessel function.

41. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Time-Averaged Shadow Moiré Method for Studying Vibrations"--November 1976.
Time-averaged shadow moiré permits the determination of the amplitude distribution of the deflection of a steady vibrating plate.

42. J. Buitrago and A. J. Durelli, "On the Interpretation of Shadow-Moiré Fringes"--April 1977.
Possible rotations and translations of the grating are considered in a general expression to interpret shadow-moiré fringes and on the sensitivity of the method. Application to an inverted perforated tube.

43. J. der Hovanesian, "18th Polish Solid Mechanics Conference." Published in European Scientific Notes of the Office of Naval Research, in London, England, Dec. 31, 1976.
Comments on the planning and organization of, and scientific content of paper presented at the 18th Polish Solid Mechanics Conference held in Wisla-Jawornik from September 7-14, 1976.

44. A. J. Durelli, "The Difficult Choice,"--May 1977.
The advantages and limitations of methods available for the analyses of displacements, strain, and stresses are considered. Comments are made on several theoretical approaches, in particular approximate methods, and attention is concentrated on experimental methods: photoelasticity, moiré, brittle and photoelastic coatings, gages, grids, holography and speckle to solve two- and three-dimensional problems in elasticity, plasticity, dynamics and anisotropy.

45. C. Y. Liang, Y. Y. Hung, A. J. Durelli and J. D. Hovanesian, "Direct Determination of Flexural Strains in Plates Using Projected Gratings,"--June 1977.
The method requires the rotation of one photograph of the deformed grating over a copy of itself. The moiré produced yields strains by optical double differentiation of deflections. Applied to projected gratings the idea permits the study of plates subjected to much larger deflections than the ones that can be studied with holograms.

46. A. J. Durelli, K. Brown and P. Yee, "Optimization of Geometric Discontinuities in Stress Fields"--March 1978.
The concept of "coefficient of efficiency" is introduced to evaluate the degree of optimization. An ideal design of the inside boundary of a tube subjected to diametral compression is developed which decreases its maximum stress by 25%, at the time it also decreases its weight by 10%. The efficiency coefficient is increased from 0.59 to 0.95. Tests with a brittle material show an increase in strength of 20%. An ideal design of the boundary of the hole in a plate subjected to axial load reduces the maximum stresses by 26% and increases the coefficient of efficiency from 0.54 to 0.90.

47. J. D. Hovanesian, Y. Y. Hung and A. J. Durelli, "New Optical Method to Determine Vibration-Induced Strains With Variable Sensitivity After Recording"--May 1978.
A steady-state vibrating object is illuminated with coherent light and its image is slightly misfocused in the film plane of a camera. The resulting processed film is called a "time-integrated specklegram." When the specklegram is Fourier filtered, it exhibits fringes depicting derivatives of the vibrational amplitude. The direction of the spatial derivative, as well as the fringe sensitivity may be easily and continuously varied during the Fourier filtering process. This new method is also much less demanding than holographic interferometry with respect to vibration isolation, optical set-up time, illuminating source coherence, required film resolution. etc.

48. Y. Y. Hung and A. J. Durelli, "Simultaneous Determination of Three Strain Components in Speckle Interferometry Using a Multiple Image Shearing Camera,"--September 1978.
This paper describes a multiple image-shearing camera. Incorporating coherent light illumination, the camera serves as a multiple shearing speckle interferometer which measures the derivatives of surface displacements with respect to three directions simultaneously. The application of the camera to the study of flexural strains in bent plates is shown, and the determination of the complete state of two-dimensional strains is also considered. The multiple image-shearing camera uses an interference phenomena, but is less demanding than holographic interferometry with respect to vibration isolation and the coherence of the light source. It is superior to other speckle techniques in that the obtained fringes are of much better quality.

49. A. J. Durelli and K. Rajaiah, "Quasi-square Hole With Optimum Shape in an Infinite Plate Subjected to In-plane Loading"--January 1979. This paper deals with the optimization of the shape of the corners and sides of a square hole, located in a large plate and subjected to in-plane loads. Appreciable disagreement has been found between the results obtained previously by other investigators. Using an optimization technique, the authors have developed a quasi-square shape which introduces a stress concentration of only 2.54 in a uniaxial field, the comparable value for the circular hole being 3. The efficiency factor of the proposed optimum shape is 0.90. whereas the one of the best shape developed previously was 0.71. The shape also is developed that minimizes the stress concentration in the case of biaxial loading when the ratio of biaxiality is 1:-1.

50. A. J. Durelli and K. Rajaiah, "Optimum Hole Shapes in Finite Plates Under Uniaxial Load,"--February 1979.
This paper presents optimized hole shapes in plates of finite width subjected to uniaxial load for a large range of hole to plate widths (D/W) ratios. The stress concentration factor for the optimized holes decreased by as much as 44% when compared to circular holes. Simultaneously, the area covered by the optimized hole increased by as much as 26% compared to the circular hole. Coefficients of efficiency between 0.91 and 0.96 are achieved. The geometries of the optimized holes for the D/W ratios considered are presented in a form suitable for use by designers. It is also suggested that the developed geometries may be applicable to cases of rectangular holes and to the tip of a crack. This information may be of interest in fracture mechanics.

51. A. J. Durelli and K. Rajaiah, "Determination of Strains in Photoelastic Coatings,"--May 1979
Photoelastic coatings can be cemented directly to actual structural components and tested under field conditions. This important advantage has made them relatively popular in industry. The information obtained, however, may be misinterpreted and lead to serious errors. A correct interpretation requires the separation of the principal strains and so far, this operation has been found very difficult. Following a previous paper by one of the authors, it is proposed to drill small holes in the coating and record the birefringence at points removed from the edge of the holes. The theoretical background of the method is reviewed; the technique necessary to use it is explained and two applications are described. The precision of the method is evaluated and found satisfactory in contradiction to information previously published in the literature.

52. A. J. Durelli and K. Rajaiah, "Optimized Inner Boundary Shapes in Circular Rings Under Diametral Compression,"--June 1979.
Using a method developed by the authors, the configuration of the inside boundary of circular rings, subjected to diametral compression, has been optimized, keeping cleared the space enclosed by the original circular inside boundary. The range of diameters studied was $0.33 \leq ID/OD \leq 0.7$. In comparison with circular rings of the same ID/OD, the stress concentrations have been reduced by about 30%, the weight has been reduced by about 10% and coefficients of efficiency of about 0.96 have been attained. The maximum values of compressive and tensile stresses on the edge of the hole, are approximately equal, there are practically no gradients of stress along the edge of the hole, and sharp corners exhibit zero stress. The geometries for each ID/OD design are given in detail.

LIGHTER AND STRONGER

INTRODUCTION

In a simplified way, the development of the study of stress distributions in elastic materials can be reviewed by saying that it started with the determination of uniaxial, uniform states of stress (as for straight columns), continued with the study of discontinuities in very large members (as in the Kirsch problem), proceeded with the parametric study of discontinuities in finite plates (as in Howland problem and in Neuber's graphs) and ends now with the determination of optimum shapes, producing minimum stress values.

There is an essential change in the last step. The purpose is no more the determination of the stress associated with a prescribed geometry, but the determination of a geometry that will satisfy prescribed conditions. This last step means a change in design procedures. The design is no more conducted by analyzing shapes, and changing them, as a consequence of the analysis, until an acceptable shape is obtained, but by shaping a body that will give the desired shape. Actually, as it will be seen later, it may be more correct to say that the design and the analysis are conducted simultaneously.

A review of some of the previous contributions to the study of the problem has been made in (1). The essential points of the proposed approach are presented in (1) and in (2) where unusual aspects of the research development were brought up. The result of the optimization of the inside boundary of a tube subjected to diametral load, for all the range of OD/ID has been presented in (3). The result of the optimization of a quasi-square hole in a plate subjected to axial load, for all the range of ratios of width of plate to width of hole has been presented in (4) and (5).

A general review of the proposed method and of the results obtained so far will be presented in what follows, avoiding unnecessary repetitions, and some new applications will be presented.

THE OPTIMIZATION PROBLEM

Consider a circular hole in a finite plate under uniaxial tension as an illustration of the subject to be dealt with in this paper. The stress distribution around the hole is characterized by a stress concentration in the neighborhood of the transverse axis, high stress gradients, poor utilization of material around the hole, and a reversal of the sign of the stresss in the neighborhood of the longitudinal axis (Fig. 1). Several methods, such as reinforcement of the edge of the hole[6], introduction of secondary holes[7], enlargment of the plate near the hole[8] etc. have been proposed to reduce the stress concentration. Methods like the variable thickness reinforcement of the edge have also been adopted to obtain more uniform stress distribution around the periphery of the hole[9]. All these methods involve either addition of more material to decrease stress concentration and to get more uniform stress distribution or introduction of another discontinuity with only the objective of reducing the stress concentration. However, there does not seem to be available any method which provides optimum shapes for discontinuities in stressed fields, and that can simultaneously 1) reduce the stress concentration, 2) obtain a uniform stress distribution around the discontinuity even in the presence of reversal of stress around the hole, and 3) decrease the weight of the plate. It is proposed that changing the shape of the discontinuity can achieve this situation and photoelasticity can be used as a tool for this purpose.

It is well known in literature that a circular hole in a circular plate is an optimum shape when subjected to a uniform pressure either on the inner or outer boundary or both (Fig. 2). Some time back, it was shown by the first author that for a hole in a biaxial stress field, a uniform stress at all points on the boundary is achieved when the hole is of elliptical

shape and the major and minor axes are in the ratio of the principal stresses, all axes being parallel and perpendicular to one another[10]. Such a hole was found to be of an optimum shape (Fig. 3). Optimization of the field, however, was not the original objective of the study of these problems, important in their own way. The same results for the elliptic holes were arrived at recently from analytical considerations by Bjorkman and Richards[11]. In these cases, there is no reversal of stress at the hole boundary.

Optimization of the shape of hole discontinuities in general stress fields, with the exception of Heywood[12],[13], does not appear to have attracted the attention of engineers and scientists till recently. This could be due to the difficulties involved in the analytical and experimental techniques in optimizing boundaries subjected to stresses of opposite sign. Heywood[13] attempted to arrive at optimum shapes for holes and fillets from large deformation experiments on thin rubber models subjected to tensile loads. Based on this study, he arrived at a barrel-shaped configuration as optimum for a hole in an infinite plate under tension. However, his technique is not general. It can be used only for tensile load applications and satisfaction of specified constrains does not seem possible. There is no proof either that the large deformed shape corresponds to the optimum shape in infinitesimal deformations.

OPTIMIZATION PROCESS

Sometime ago, the first author used the concept of an ideal fillet, defined it as a fillet without stress concentration and related it photoelastically to the coincidence of the boundary with an isochromatic fringe. Some references can be found in a book [14], reports and early papers[15],[16],[17

The same concept can be extended to the optimization of hole shapes with the basic difference that on the boundary of a hole, stress can change sign depending on the applied loading and the shape of the hole. Optimization has to be conducted for both the tensile and compressive segments of the hole boundary. The optimization process involves the removal of material from the low stress portions of the hole boundary of a photoelastic model till an isochromatic fringe coincides with the boundary both on the tensile and compressive segments.

Following the above approach, optimization of hole shapes was carried out in finite and infinite plates under uniaxial tension [(1),(4)], in infinite plates under equal biaxial loading of opposite sign [(5)], in circular rings under diametral compression [(3)] and in plates under bending. Further work on the optimization of external boundaries in finite plates with optimized holes and under uniaxial tension has also been carried out.

METHOD USING PHOTOELASTICITY

The basic method of optimization using photoelastic models has already been explained in [(1),(3),(4)] and [(5)]. Here, some technical aspects will be emphasized

Models are made of materials easy to machine and relatively free from time-edge effects. Homalite-100 would appear to be the most suitable. It should be emphasized that it would be difficult to use this method in a lens polariscope where the field is small and the model cannot be seen directly through the analyzer at the time the operator works on it. A large field diffused light polariscope is ideal for this purpose. Sometimes it may be practical to wear a binocular magnifier with a set of polarizer and quarter-wave plates attached to each of its lenses.

The model should be as large as possible and the operator should be able to remove material conveniently from the edges, particularly at the corner regions. Material from the low stress regions around the hole or fillet

is removed after the model has been loaded. When a small amount of material has to be removed from the edges, a good filing set, such as a tool and die makers file, is required. For removal of large amounts of material, a portable light weight router can be used.

The transformation of the shape from the original design to the optimum design can be done in a shorter or longer time depending on the length of the low stress region involved. As the operator removes material by filing off zones of low stress, the order of the fringe at the zone of high stress decreases. When both its inner and outer boundaries are to be optimized, due to the mutual dependency of the stresses on the two boundaries, it may be necessary to work on both the boundaries alternately. In a large field diffused light polariscope,the operator can watch the transferring of fringes as the filing takes place until the moment when one single fringe coincides with the boundary of the model. The improved design always brings the stress concentration factor (s.c.f.) down, reduces the weight of the component and increases the strength.

CRITERIA

The definition of the problem requires the specification of the constrains imposed by the design. These constrains dictate the maximum amount of material that may be removed from the low stress regions. If the functional requirements permit appreciable changes in design, one can reach several answers to the optimization problem.

It was proposed in an earlier paper[(1)] and used in others[(3),(4)] and [(5)], that the degree of optimization be evaluated quantitatively by a coefficient of efficiency.

$$k_{eff} = \frac{1}{S_2-S_0}\left\{\frac{\int_{S_0}^{S_1}\sigma_t^+ ds}{\sigma_{a\ell\ell}^+} + \frac{\int_{S_1}^{S_2}\sigma_t^- ds}{\sigma_{a\ell\ell}^-}\right\}$$

where $\sigma_{a\ell\ell}$ represents the maximum allowable stress (the positive and negative superscripts referring to tensile and compressive stresses, respectively), S_0 and S_1 are the limiting points of the segment of boundary subjected to tensile stresses and S_1 and S_2 are the limiting points of the segment of boundary with compressive stresses.

The coefficient of efficiency k_{eff} shows how efficiently the material at the hole boundary has been utilized for the given field. k_{eff} equal to one would mean that the stress levels are constant both on the tensile and compressive regions around the hole. The closer k_{eff} is to unity, the more efficient the design is. The coefficient indicates also how far the design is from the limit beyond which the design cannot be further improved.

The above criterion will be used to evaluate the different optimized hole shapes and fillets presented here.

USE OF OTHER STRESS ANALYSIS METHODS

In principle, it is possible to use other methods than photoelasticity to optimize the geometry of holes and fillets in stress fields. Figure 18 is an example of the isopachic pattern in the optimized ring. Since isochromatics and isopachics coincide at free boundaries, the same approach described above can be followed. However, the idea would not be practical since it is appreciably more difficult to work with the holographic set up than with the polariscope

It was pointed out previously[18], that the distance between a free boundary and an immediate isostatic is inversely proportional to the value of the stress at the boundary. Figure 4 shows the isostatic pattern obtained using brittle coatings, along a fillet contour. This property would require that the first isostatic,

next to the optimized boundary, be parallel to it. Again, although it is possible to use this property, the idea is not practical since a new brittle coating test should be conducted after each change in shape.

It is also possible to use finite element methods to optimize configurations of plates loaded in their plane. The relative merit of this approach and the approach used here has been discussed elsewhere[19].

APPLICATIONS

The optimization method will now be illustrated with several applications and some of the important conclusions will be highlighted.

Hole in a Large Plate: Uniaxial Loading

The isochromatic pattern for an optimized hole of width D in a large plate of width W under uniaxial loading (D/W = 0.14) is presented in Fig. 5. The hole has a s.c.f. of only 2.54 compared to 3 for a circular hole leading to a 15% reduction in s.c.f. (Unless stated otherwise, s.c.f. are referred to the stress averaged over the net area. In this case, the figure 3, referred to the gross area, is probably convenient to designers).

Hole in a Large Plate: 1 : -1 Biaxial Loading

The isochromatic pattern for a double-barrel shaped hole in a large plate (D/W = 0.16) under biaxial loading of equal and opposite sign is shown in Fig. 6. In this case, there is a 10% reduction in s.c.f. compared to a circular hole.

Hole in a Finite Plate: Uniaxial Loading

The isochromatic patterns for two typical optimized holes in a finite plate under uniaxial loading are shown in Fig. 7 and 8. The stress distributions around the optimized holes and the corresponding circular holes are shown in Fig. 9 and the s.c.f. in the two cases along with percentage increase in hole area and k_{eff} are given in Fig. 10. It is seen that the s.c.f. have been

significantly reduced in the case of optimized holes with the largest decrease occuring for large holes. Simultaneously, the hole size also has increased significantly leading to a lighter plate. The increase in strength/weight ratio is quite enormous. The optimized hole geometries have been fitted with straight lines and arcs of circles and the information is given in Figs. 11 and 12. The information obtained for a square hole could be extended for rectangular holes for certain ranges of hole sizes and is shown in Fig. 13.

It is interesting to note that sharp corners are occuring in optimized holes with only zero stress at those points.

Hole in a Circular Ring under Diametral Compression

The isochromatic patterns for two typical optimized holes in a circular ring under diametral compression are shown in Figs. 14 and 15. The stress distribution around the optimized and the corresponding circular holes are presented in Fig. 16. The information on s.c.f. weight and k_{eff} are given in Fig. 17. Once again, it is seen that the increase in strength/weight ratio is substantial for the optimized ring. The k_{eff} is about 0.95 in all optimized rings while it is only about 0.61 in all circular rings. The isopachic pattern for an optimized ring with ID/OD = 0.7 is presented in Fig. 18. It is seen that isopachics also follow the optimized boundaries as may be expected. For this ID/OD ratio, the external boundary has also become an optimum boundary as the inner boundary became an optimum. Once again, the optimized hole geometries were fitted with straight lines and arcs of circles and the data is shown in Fig. 19.

Fracture tests were also conducted on the optimized and circular rings and a typical fracture pattern is in Fig. 20. Fracture in the optimized ring does not start at the hinges. The material used in this case was the polymer Homalite 100.

INDUSTRIAL APPLICATIONS

Three attempts have been made at optimizing holes and fillets for industrial applications: one to improve the attachment of a turbine blade to the turbine rotor, and two to improve the star-shape configuration of solid propellant grains. In what follows, these attempts will be evaluated using the coefficient of efficiency as a criterium, and two more related applications will be described.

a) Dove-tail

The stress distribution on the fillet of the blade is shown in Fig. 21. There is an appreciable decrease in the s.c.f., and an increase in the coefficient of efficiency. However, the coefficient of efficiency is only 0.76 suggesting that the fillet shape could be further changed to approach the optimum.

An attempt was also made to optimize the slot of the turbine rotor part of the blade dove-tail attachment. Two loading conditions were considered, a radial pulling (corresponding to the centrifugal force acting on the blade) and a tangential force. This type of problem is a further complication in the process of optimization since the optimum shape will not be the same for both loading conditions. The process was applied independently to each loading condition. Figure 22 shows the isochromatics for both designs, corresponding to the tangential force. The stresses so obtained, combined with those obtained in a similar manner for the case of the radial load, and the results obtained for other ratios of both loads, are shown in Fig. 23. The curve corresponding to the ratio 2.3 between tangential σ_n and radial σ_r, which was the design specification, gives uniform stress distribution along the boundary.

b) Star-shapes

Figure 24 shows a comparison between the stresses at two fillets located at the bottom of the straight side ray of a star, perforating a disk subjected to uniform pressure.

The similar proble, when the tip of the star rays was expanded with a triangular end as shown in Fig. 25, was the object of an early parametric study. Using the idea presented in [10], it was decided to give to the tip elliptic configurations. These ellipses, however, lying on a circle rather than on a straight line, had to be distorted following the procedure shown in Fig. 25. The idea seemed reasonable since the tops of the rays are essentially subjected to tangential and radial approximately uniform loadings. The parameters to be considered are: 1) the number of rays, 2) the web fraction or amount of material beyond the top of the perforation, 3) the ratio between the perimeter of the solid disk, and the perimeter of the perforation at the point of the major axis of the ellipse. The results of several tests are shown in Fig. 26, from which the width of the basic tip a and the optimum b/a can be obtained. The star-shaped configuration with angular sided rays, shown in [1] has a coefficient of efficiency of 0.91 and seems very close to optimum.

PRECISION OF THE MEASUREMENTS

From experience in previous determinations conducted using the same optical method and similar materials and loading devices, it is estimated that fractional orders of birefringence can be measured in the polariscope with a reproducibility of about $\pm$ 0.01 of a fringe. This precision can be improved when readings are repeated. Systematic errors at edges can be appreciable when measurements are taken several days after the machining of the specimens, particularly in summer when moisture content in laboratories may be high.

Most of the measurements reported were taken immediately after machining When that was not the case, the residual effect was measured independently and subtracted.

The precision increases with the level of the response. It is lower for large plates with small holes (small D/W). Since it is desirable to make holes relatively large to permit the machining of the edges with precision, the plates have to be large, the loading capacity of the frame is soon approached and the average stress is relatively low. The precision is higher for large holes (large D/W) when 5 or 6 fringe orders can easily be obtained. It is estimated that for the last case, the precision of results obtained from individual specimens was of the order of 2% or 3% of the average stress. In most of the papers to which a reference is made here, however, problems have been solved parametrically and curves have been drawn through the points corresponding to the individual tests. This operation increases appreciably the precision.

No estimate is being made of the error made when circles are matched to the experimentally obtained shapes, and their radii and centers are determined. Here also, the values have been obtained parametrically for a large range of D/W and curves drawn through the points. It is believed that little error is associated with this part of the operation.

ACKNOWLEDGMENTS

Some of the photoelastic tests and optimization of shapes related to the star perforated disk were conducted by R. Lake, and some of those related to the slot in the turbine wheel dove-tail were conducted by W. F. Riley. Some of the results reported here have been taken from research programs supported financially by the Office of Naval Research and by the National Science Foundation. Appreciation is expressed here to the monitors of the programs, N. Perrone, N. Basdekas and C. Astill. The reproduction of the manuscript was made by P. Baxter

References

1. Durelli A. J., Brown K. and Yee P., "Optimization of Geometric Discontinuities in Stress Fields," Exp. Mech., Vol. 18, pp. 303-308, 1978.
2. Durelli A. J., Rajaiah K., Hovanesian J. D. and Hung Y. Y., "General Method to Directly Design Stress-Wise Optimum Two-Dimensional Structures." To be published in Mech. Res. Comm.
3. Durelli A. J. and Rajaiah K., "Optimized Inner Boundary Shapes in Circular Rings Under Diametral Compression," ONR Report No. 52, 1979.
4. Durelli A. J. and Rajaiah K., "Quasi-Square Hole with Optimum Shape Subjected to Uniaxial Loading," To be published in J. Mech. Design.
5. Durelli A. J. and Rajaiah K., "Optimum Hole Shapes in Finite Plates Under Uniaxial Load," To be published in J. Appl. Mech.
6. Peterson R. E., "Stress Concentration Factors", Wiley, 1974.
7. Erickson P. E. and Riley W. F., "Minimizing Stress Concentrations Around Circular Holes in Uniaxially Loaded Plates", Exp. Mech., Vol. 18, pp. 97-100, 1978.
8. Coker F. G. and Filon L. N. G., "A Treatise on Photoelasticity", Cambridge University Press, 1931.
9. Dhir S. K. and Brock J. S., "A New Method of Reinforcing a Hole Effecting Large Weight Savings", Int. J. Solid Structures, Vol. 6, pp. 259-275, 1970.
10. Durelli A. J. and Murray W. M., "Stress Distribution Around an Elliptical Discontinuity in any Two-dimensional, Uniform, and Axial System of Combined Stress", Proc. Exp. Str. Analysis, Vol. 1, No. 1, pp. 19-31, 1943.
11. Bjorkman, Jr. G. S. and Richards, Jr. R., "Harmonic Holes--an Inverse Problem in Elasticity", J. Appl. Mech., Vol. 43, pp. 414-418, 1976.
12. Heywood R. N., "Designing by Photoelasticity", Chapman and Hall, 1952.

13. Heywood R. N., "Photoelasticity for Designers", Pergamon, 1969.

14. Durelli A. J. and Riley W. F., "Introduction to Photo-mechanics," Prencice Hall, p. 228, 1965.

15. Durelli A. J.,"Experimental Strain and Stress Analysis of Solid Propellant Rocket Motors," Mech. and Chem. of Solid Propellants, Pergamon, pp. 381-442, 1967.

16. Durelli A. J., Dally J. W. and Riley W. F., "Stress and Strength Studies on Turbine Blade Attachments," Proc. SESA, Vol. XVI, No. 1, pp. 171-182, Oct. 1957.

17. Durelli A. J., Parks V. J. and Uribe S., "Optimization of a Slot End Configuration in a Finite Plate Subjected to Uniformly Distributed Load," J. Appl. Mech., Vol. 35, pp. 403-406, 1968.

18. Durelli A. J., "Determination of Stresses on Free Boundaries by Means of Isostatics." Proc. Fifteenth Semi-Annual Eastern Photoelasticity Conf., June 20, 1942.

19. Durelli A. J., "Discussion of A. Francavilla, C. V. Ramakrishnan and O. C. Zienkievicz paper: "Optimization of Shape to Minimize Stress Concentration." J. Strain Analysis, Vol. 11, No. 3, pp. 191-192, 1976.

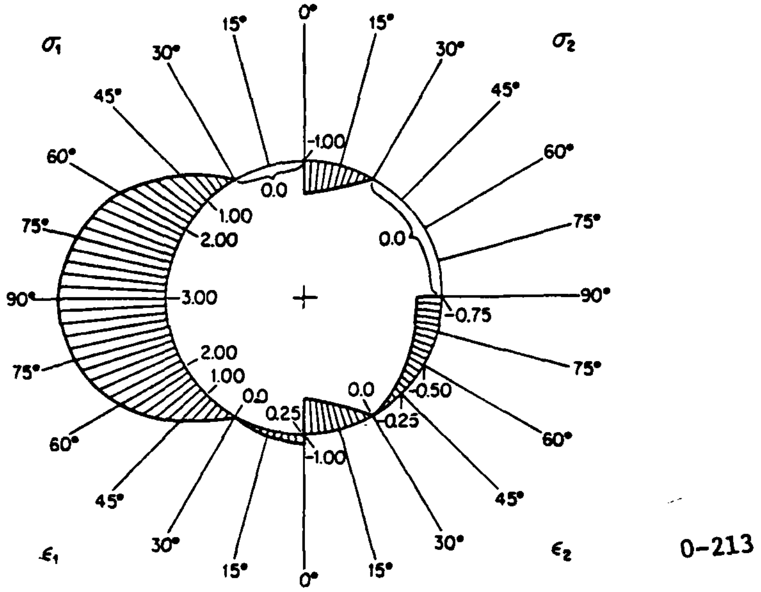
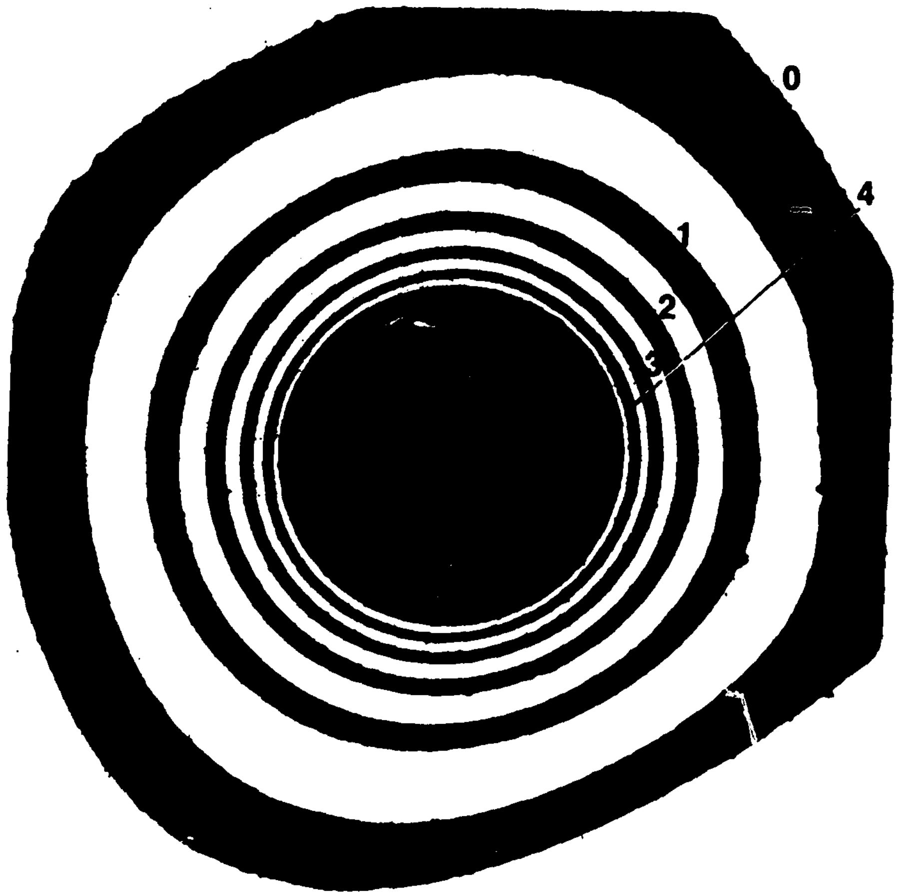


FIG. 1 DISTRIBUTION OF STRESSES AND STRAINS AT THE CIRCULAR BOUNDARY OF AN INFINITE PLATE, WITH A CIRCULAR HOLE, UNDER A UNI-DIMENSIONAL LOAD. (KIRSCH'S SOLUTION) $\nu = 0.25$.



0-212

FIG. 2 OPTIMUM SHAPE OF A HOLE IN A PLATE SUBJECTED TO IN-PLANE ISOTROPIC STATE OF STRESS. (ISOCHROMATICS)

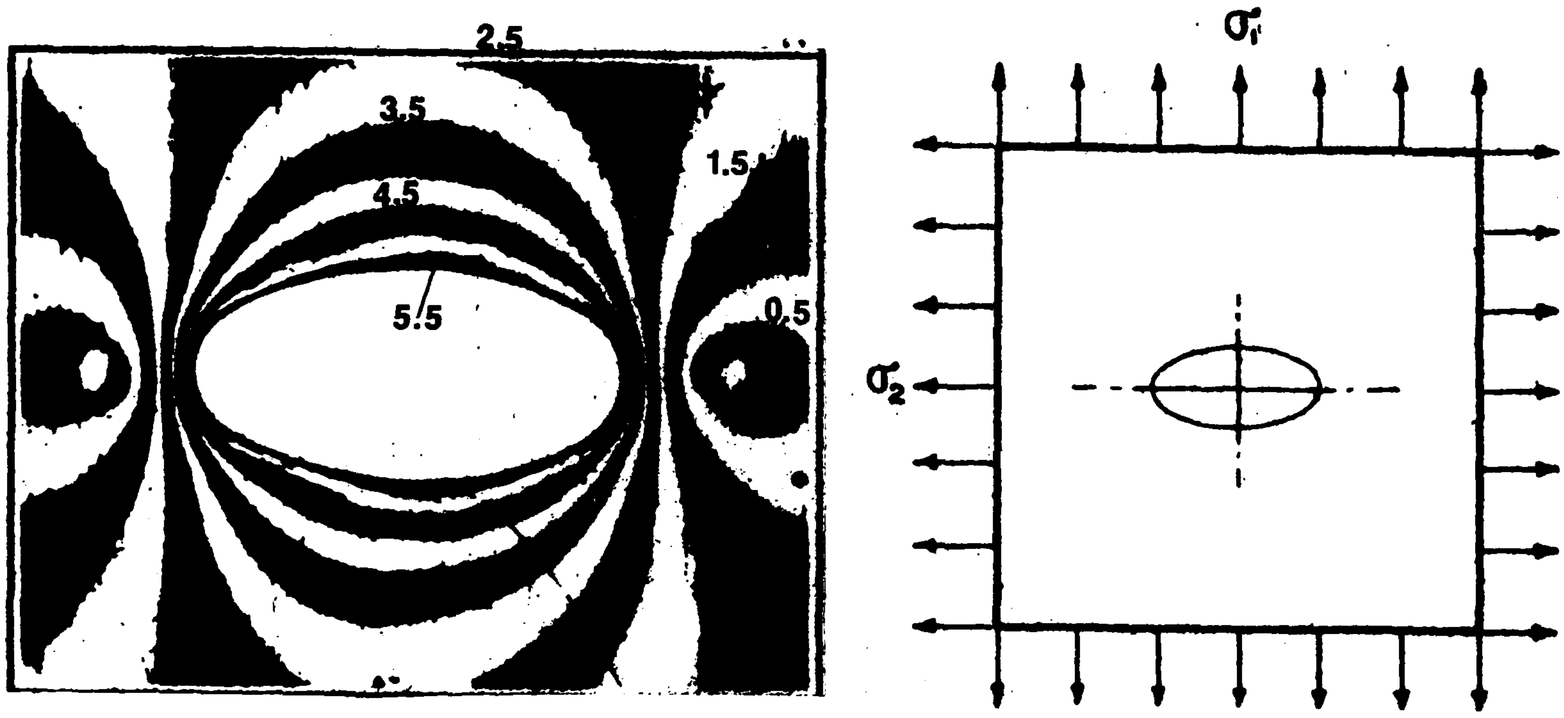


FIG. 3 THE ELLIPSE IS THE OPTIMUM SHAPE OF A HOLE IN A PLATE SUBJECTED TO IN-PLANE STRESSES WHEN THE RATIO OF THE SEMIAXES IS THE SAME AS THE RATIO OF THE PRINCIPAL STRESSES.

0.211

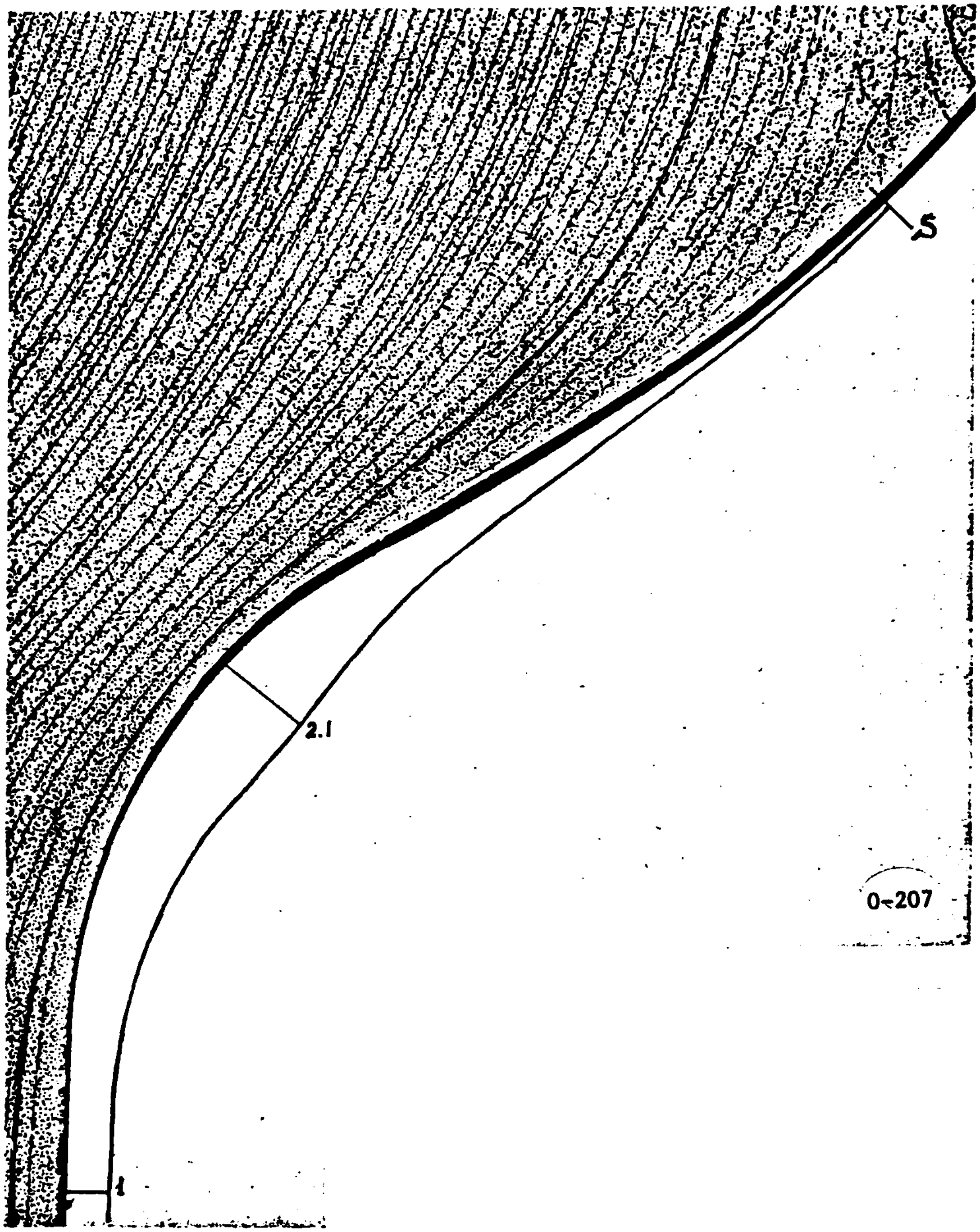


FIG. 4 DETAIL OF THE FILLET. THE INVERSE OF THE DISTANCE BETWEEN THE ISOSTATIC AND THE BOUNDARY IS PROPORTIONAL TO THE STRESS ON THE BOUNDARY. S IS THE SINGULAR POINT.

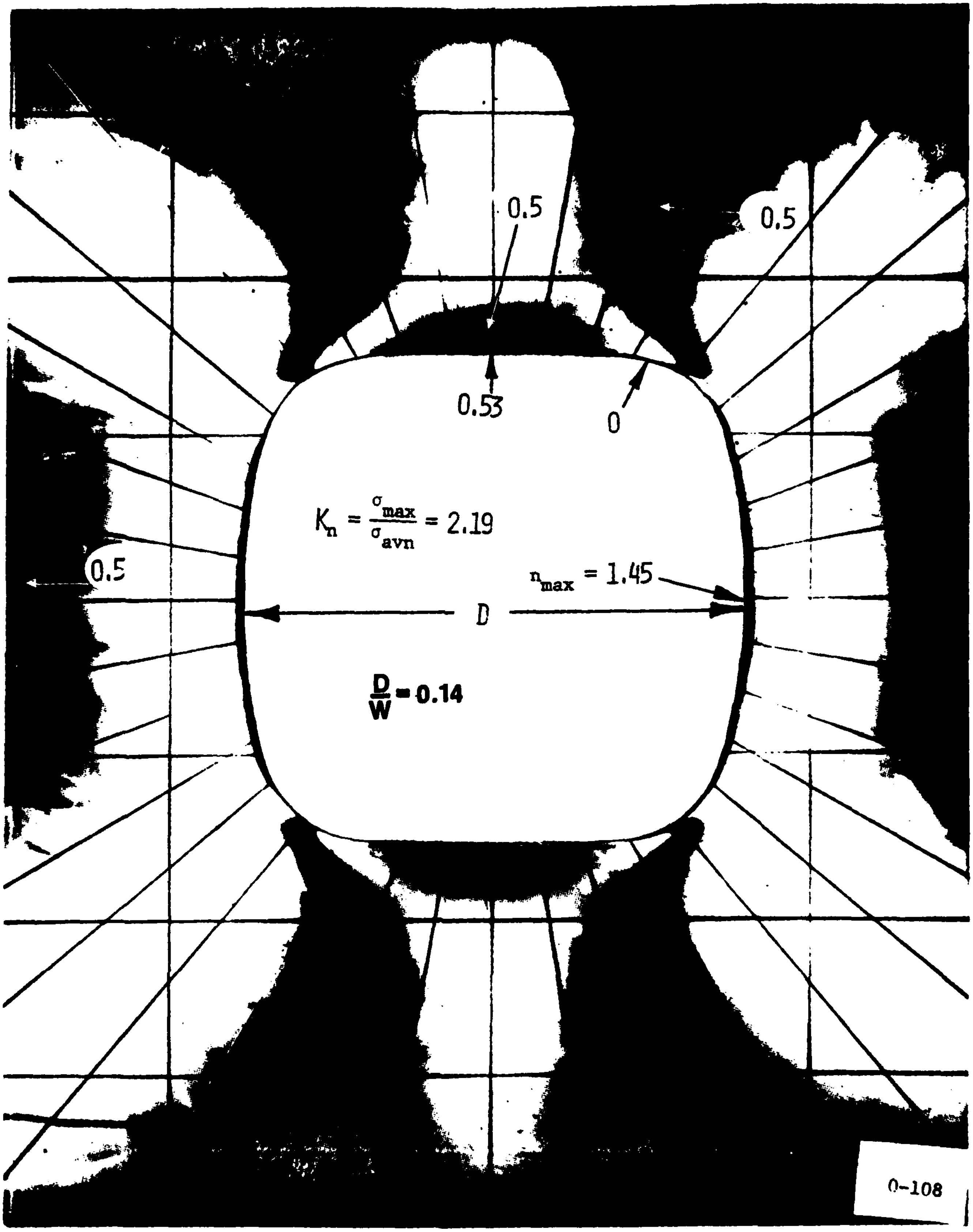


FIG. 5 STRESSES AT THE BOUNDARY OF AN OPTIMIZED QUASI-SQUARE HOLE IN LARGE PLATE ($\frac{D}{W}$ = 0.140) SUBJECTED TO UNIAXIAL LOAD

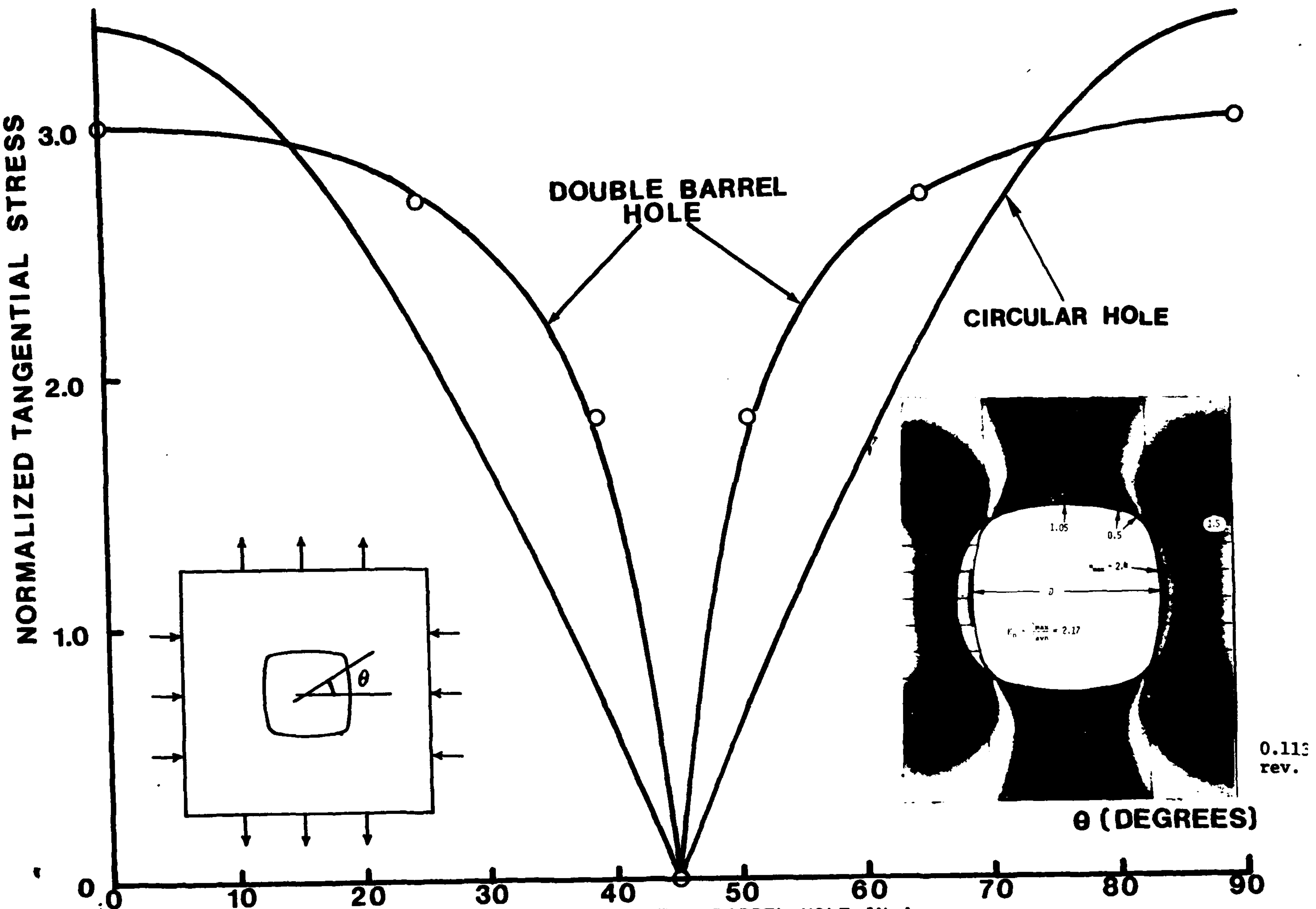


FIG. 6 DISTRIBUTION OF STRESS AROUND A DOUBLE BARREL HOLE IN A LARGE PLATE SUBJECTED TO BIAXIAL LOADING OF 1:-1 (PURE SHEAR)

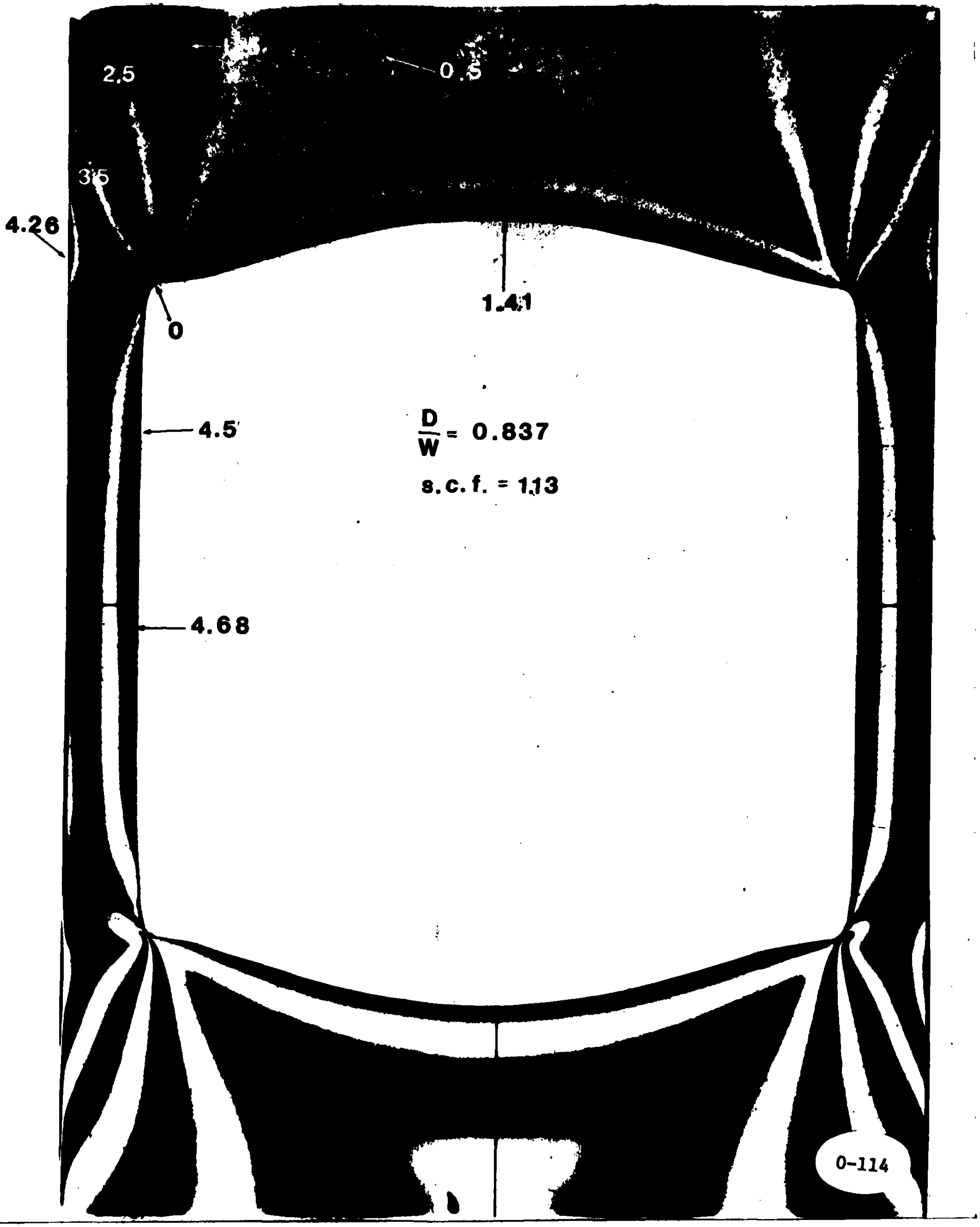


FIG. 7 OPTIMUM SHAPE OF A HOLE IN A FINITE PLATE SUBJECTED TO AXIAL LOADING (D/W = 0.837).

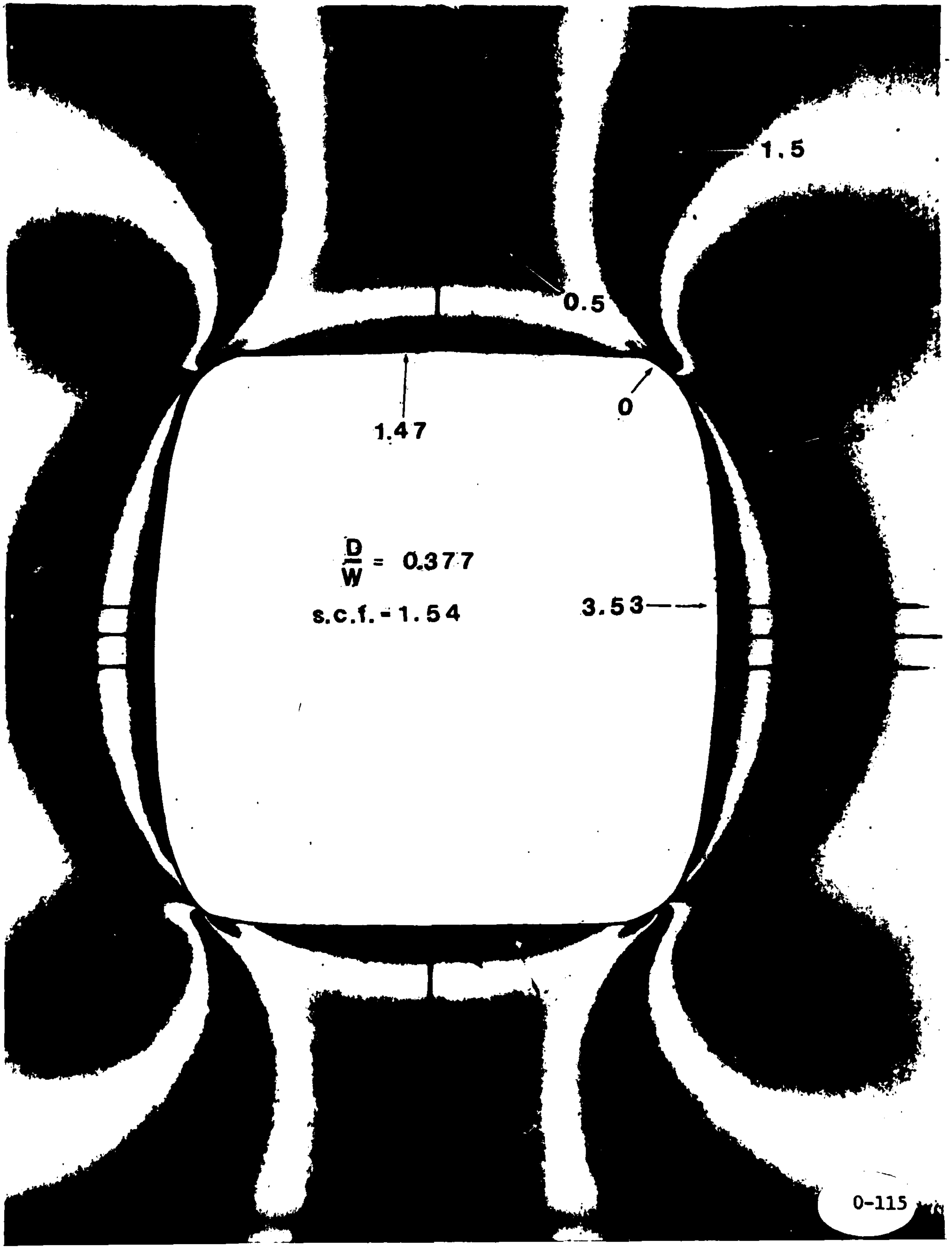


FIG. 8 OPTIMUM SHAPE OF A HOLE IN A PLATE SUBJECTED TO AXIAL LOADING (D/W = 0.377).

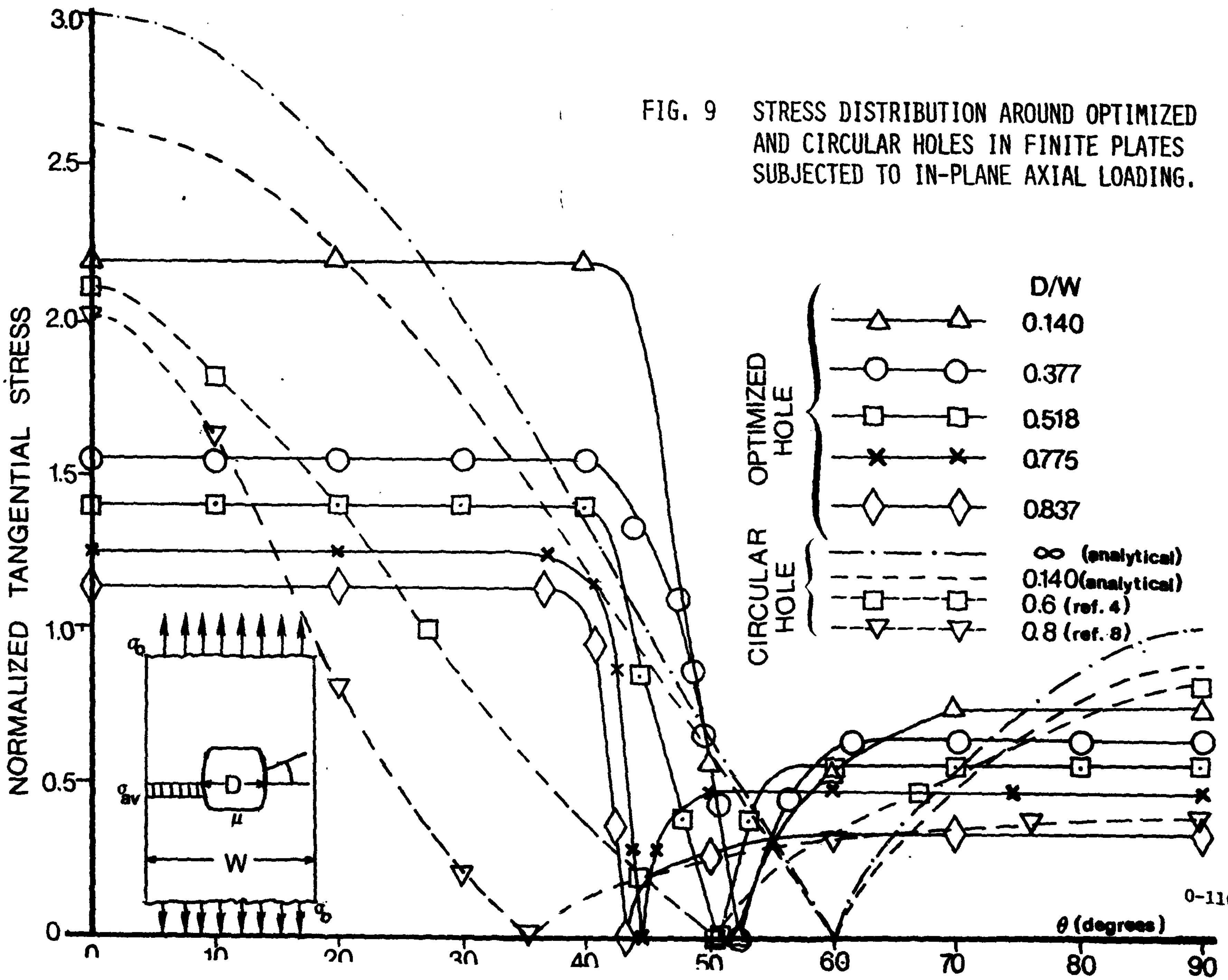


FIG. 9 STRESS DISTRIBUTION AROUND OPTIMIZED AND CIRCULAR HOLES IN FINITE PLATES SUBJECTED TO IN-PLANE AXIAL LOADING.

0-116

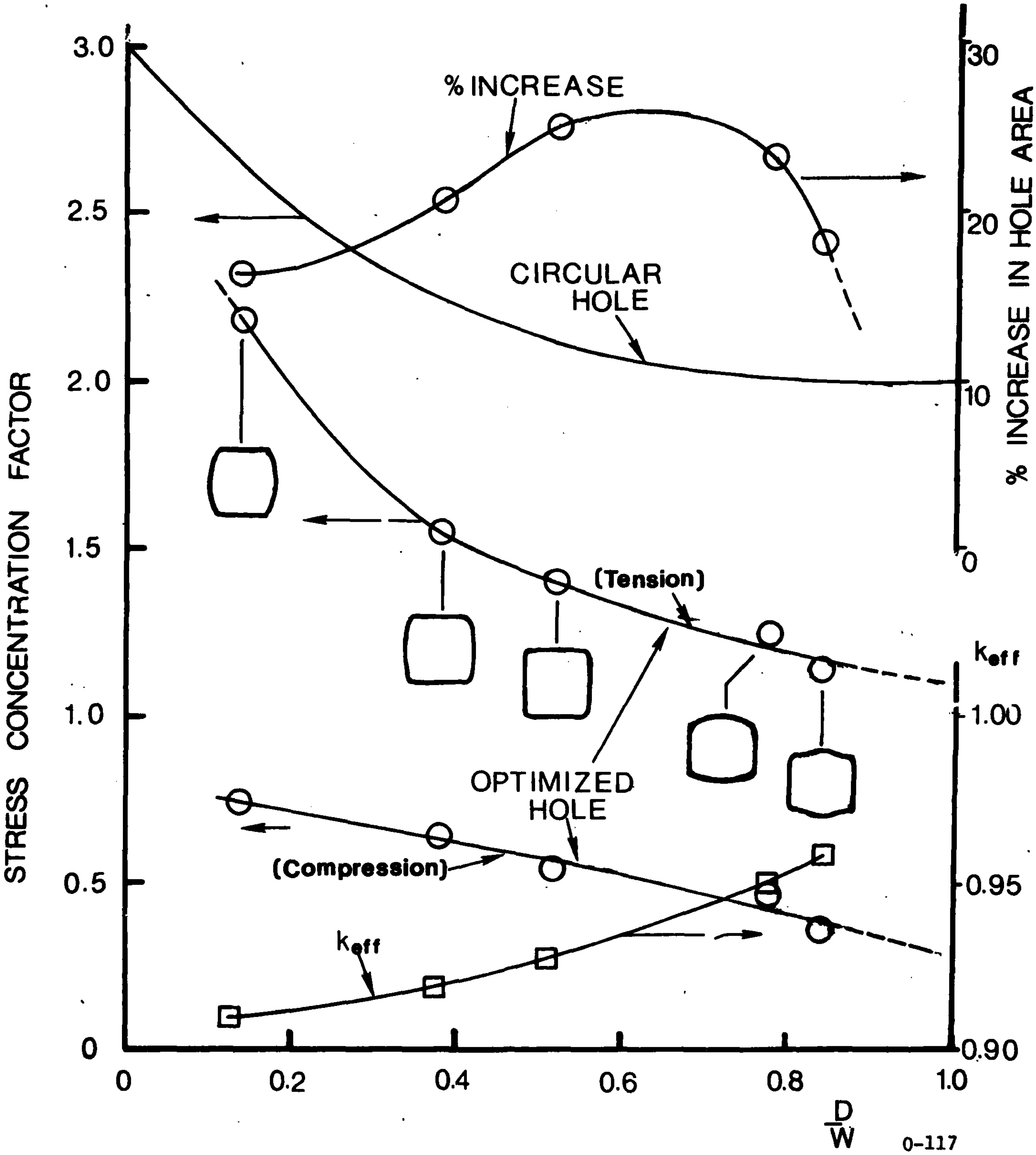


FIG. 10 STRESS CONCENTRATION FACTORS FOR OPTIMIZED HOLES IN FINITE PLATES SUBJECTED TO IN-PLANE AXIAL LOADING.

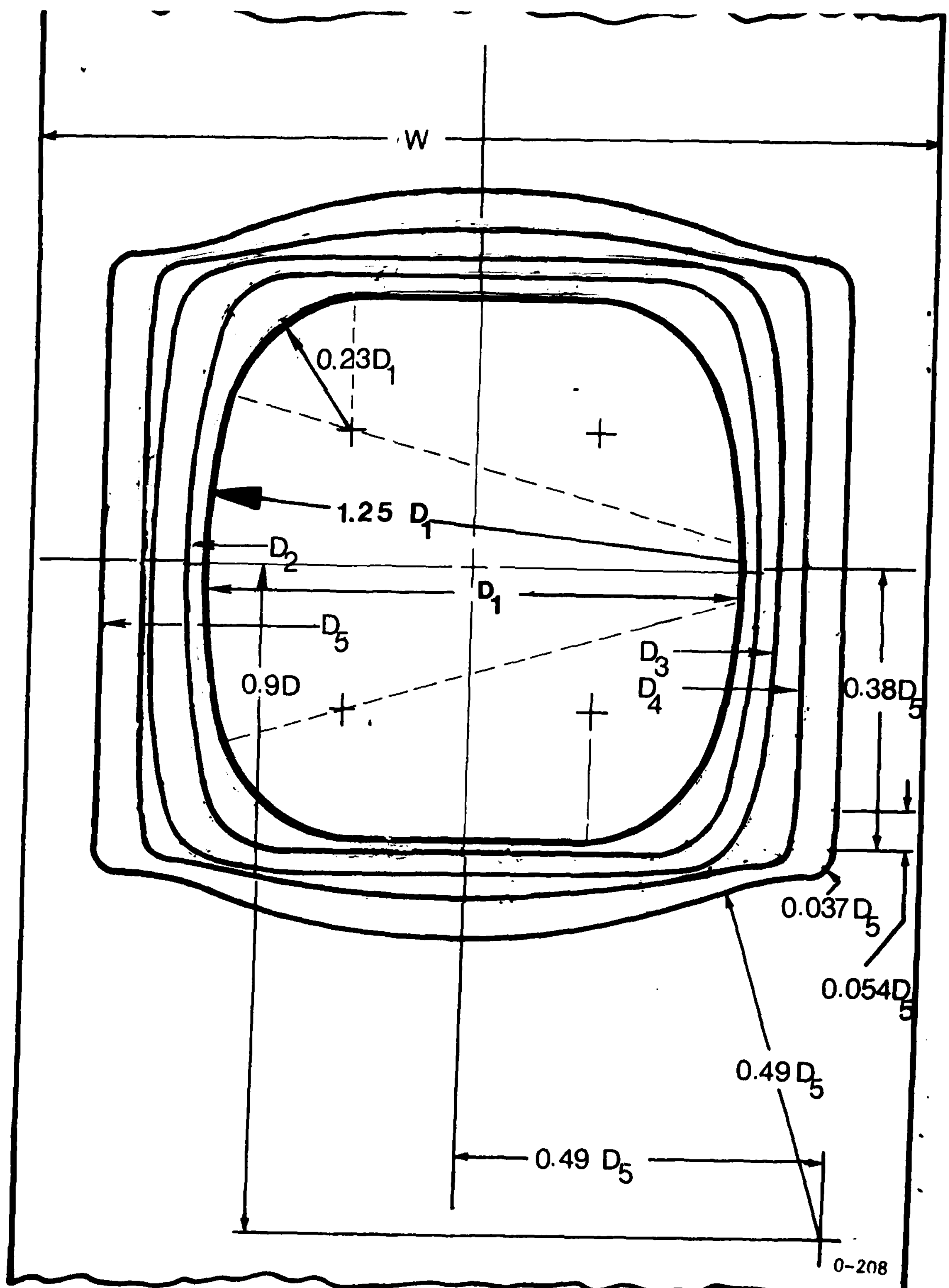
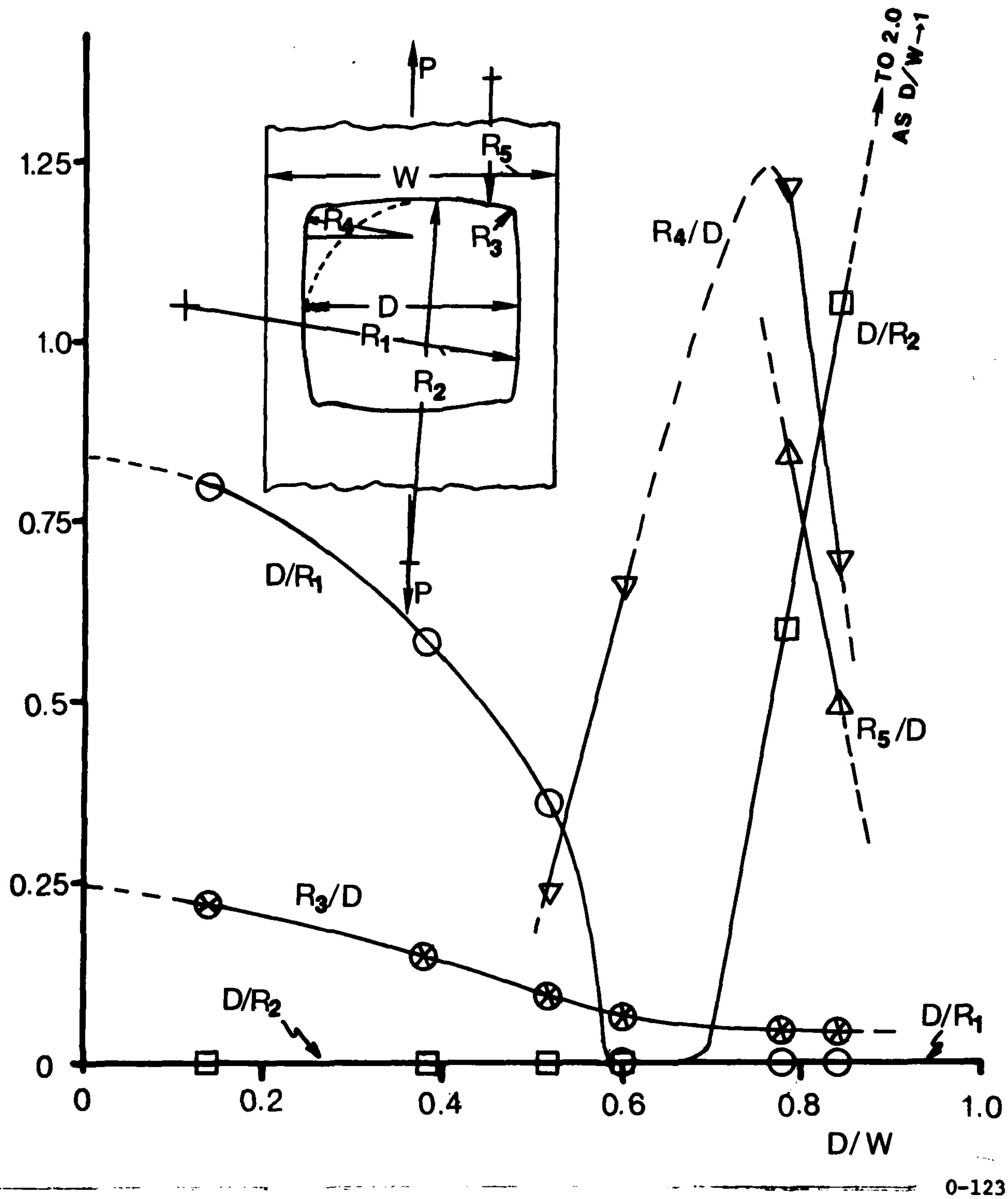


FIG. 11 OPTIMIZED GEOMETRY OF A QUASI-SQUARE HOLE IN A FINITE PLATE SUBJECTED TO IN-PLANE AXIAL LOADING (OTHER DIMENSIONS IN FIG. 10)



0-123

FIG. 12 RADII OF ELEMENTS OF THE HOLES PRODUCING OPTIMUM DISTRIBUTION OF STRESS IN FINITE PLATES, SUBJECTED TO IN-PLANE AXIAL LOADING.

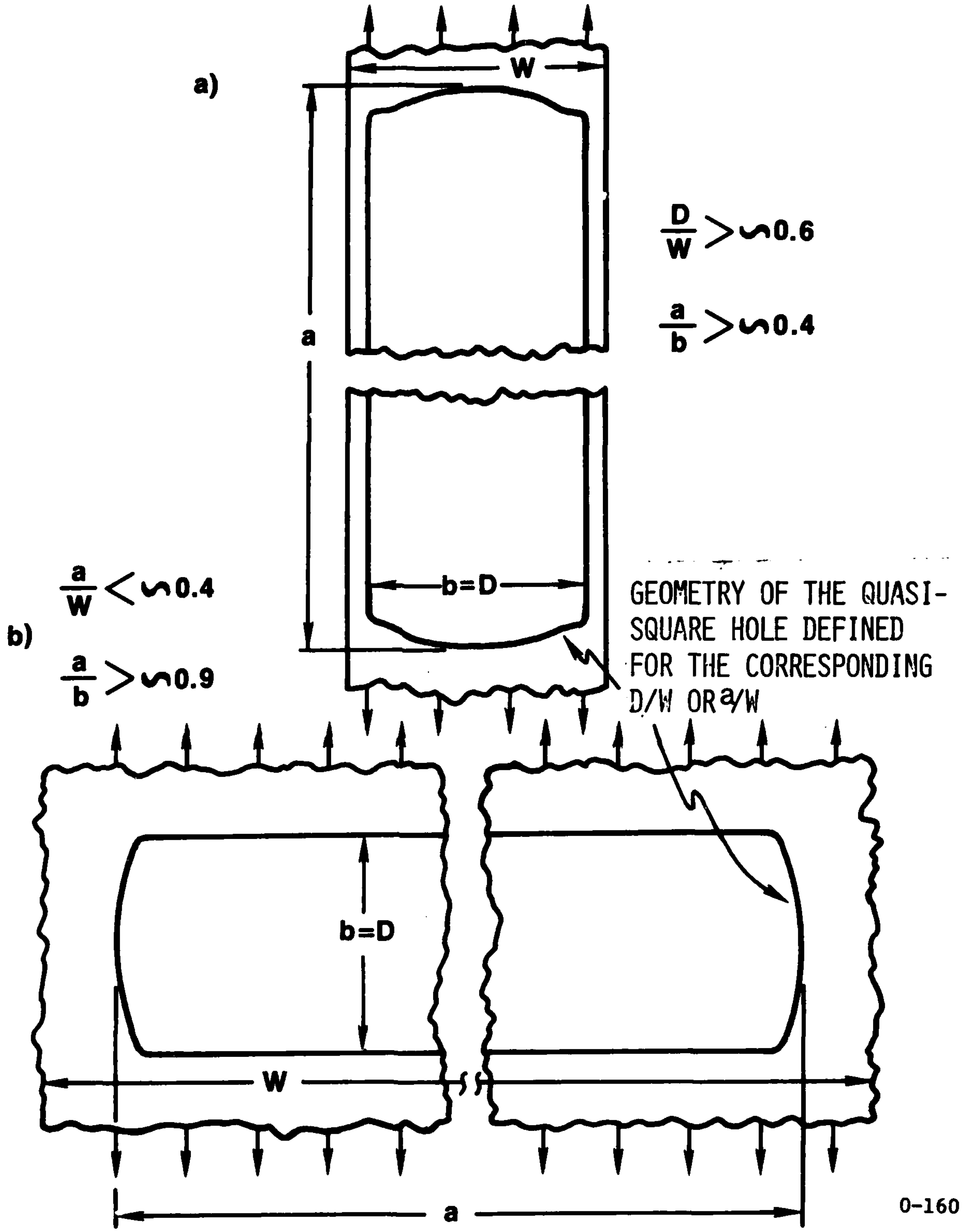


FIG. 13 OPTIMUM SHAPE FOR RECTANGULAR HOLES IN PLATES SUBJECTED TO IN-PLANE AXIAL LOADING. (GENERALIZATION OF THE QUASI-SQUARE OPTIMUM SHAPE)

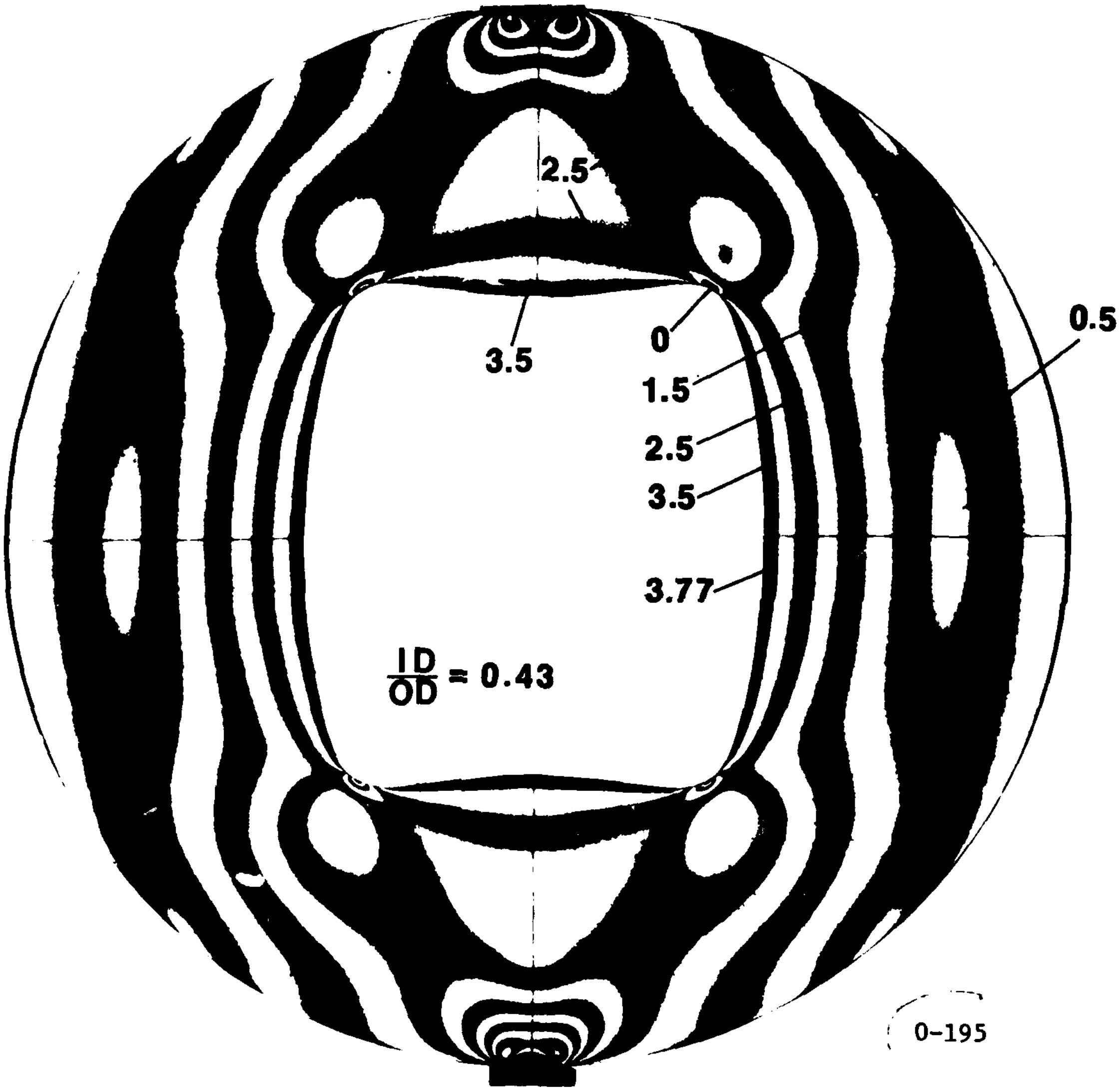


FIG. 14 ISOCHROMATICS IN A CIRCULAR RING WITH OPTIMIZED INNER BOUNDARY, WHEN SUBJECTED TO DIAMETRAL COMPRESSION ID/OD = 0.43.

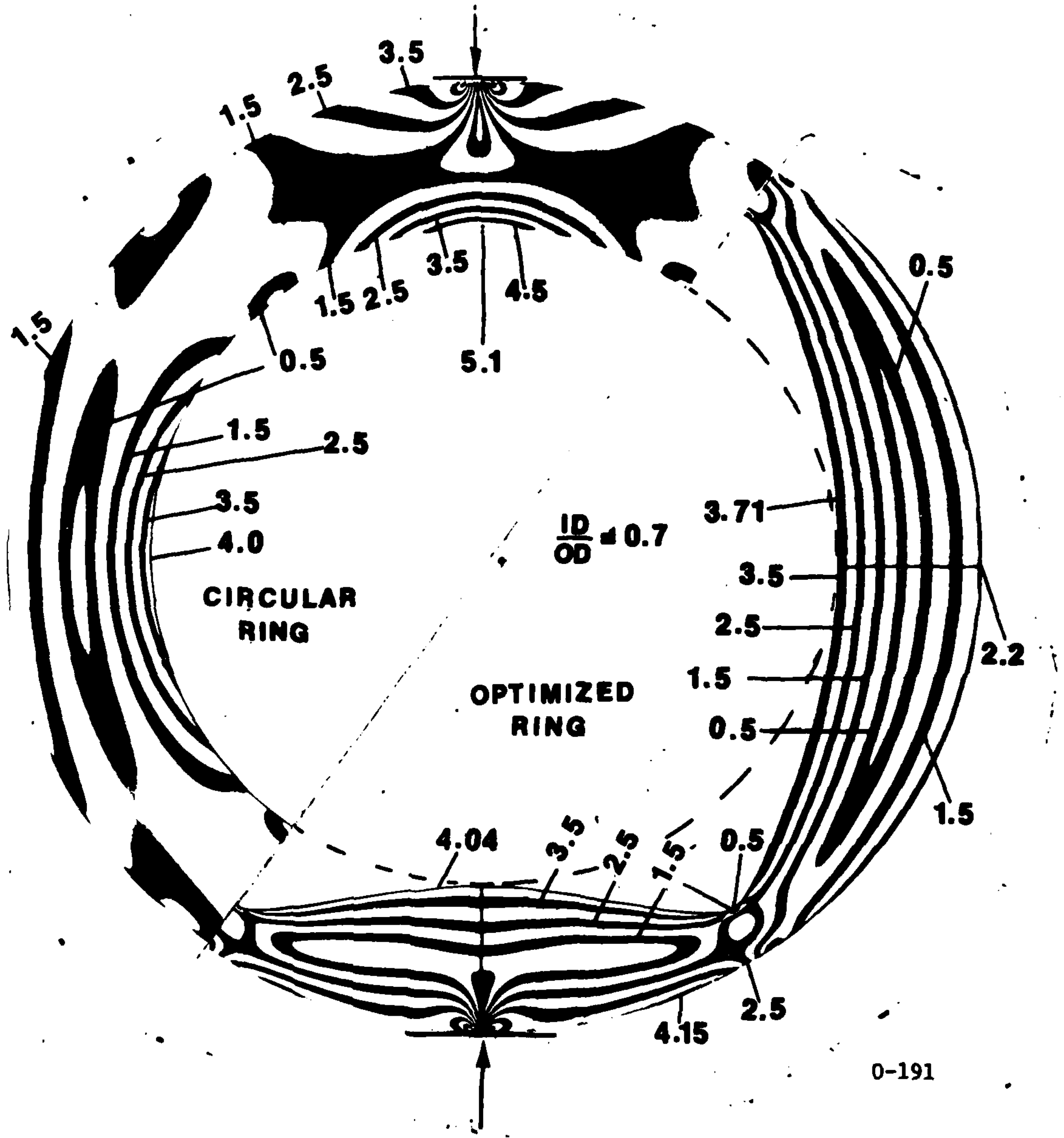


FIG. 15 LIGHT FIELD ISOCHROMATICS IN CIRCULAR AND OPTIMIZED RINGS, SUBJECTED TO THE SAME LOAD

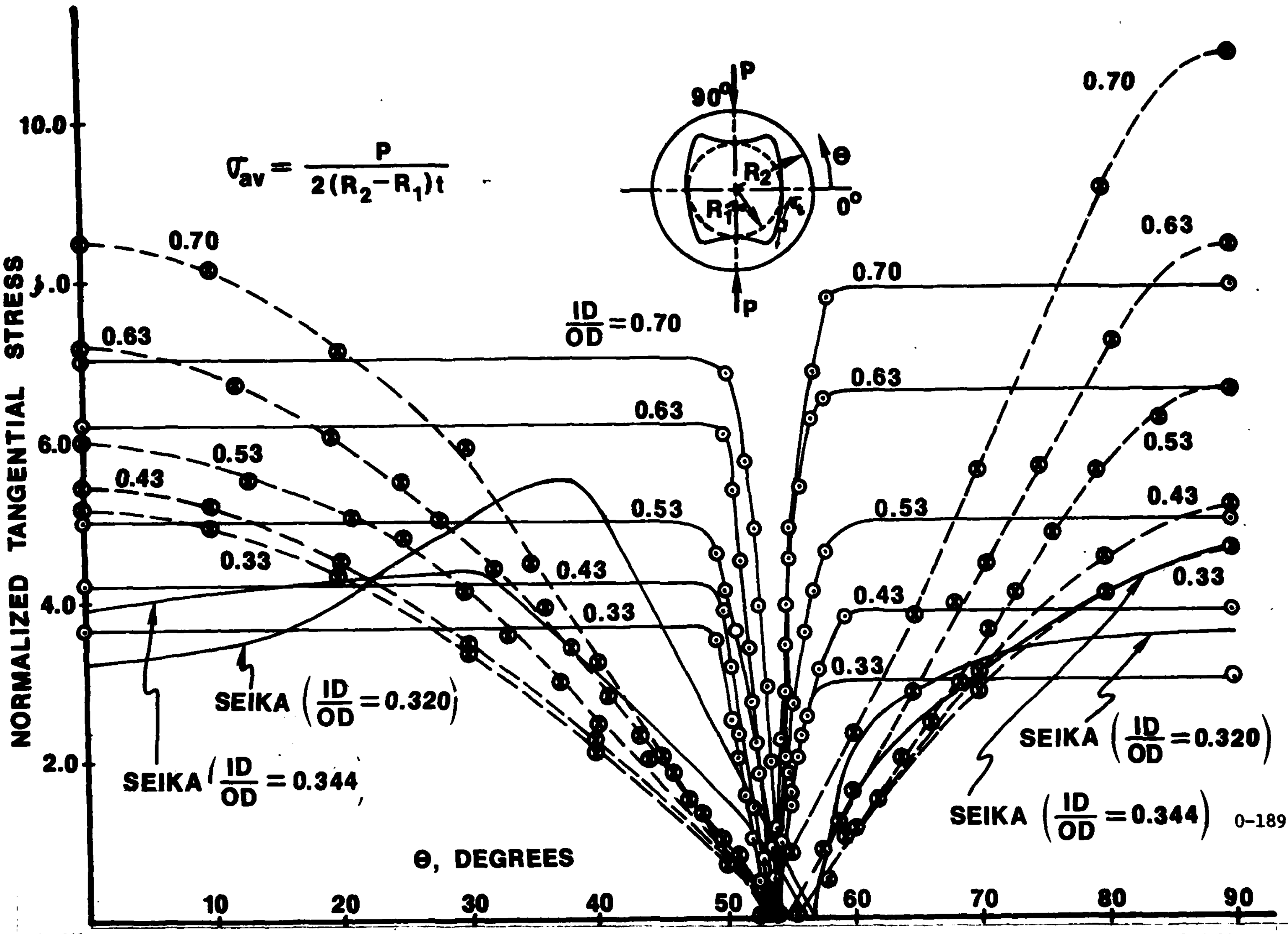
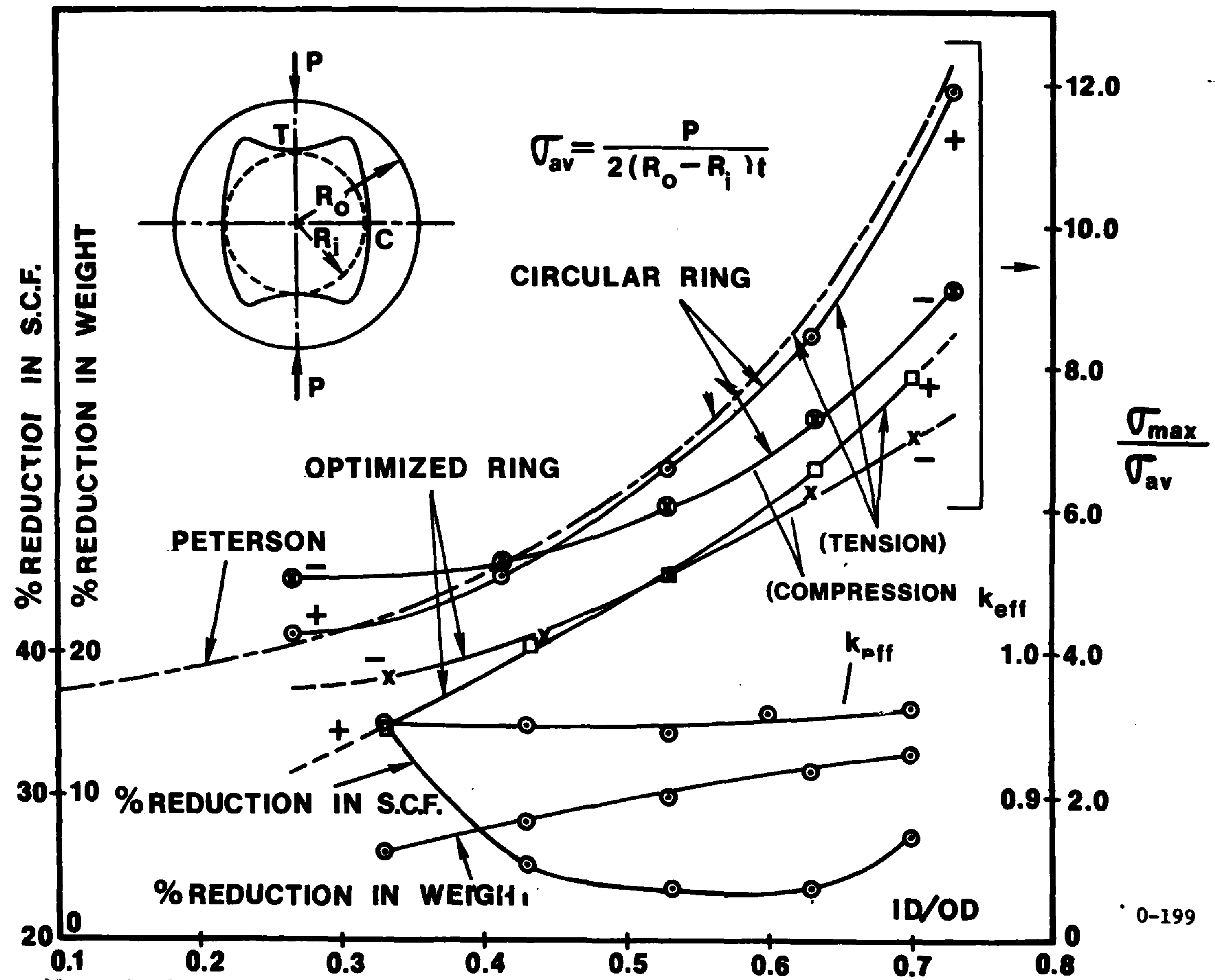


FIG. 16 STRESS DISTRIBUTION ALONG THE EDGE OF THE OPTIMIZED INNER BOUNDARY OF CIRCULAR RINGS SUBJECTED TO DIAMETRAL COMPRESSION.



0-199

FIG. 17 STRESS DISTRIBUTION ALONG THE EDGE OF THE OPTIMIZED INNER BOUNDARY OF CIRCULAR RINGS SUBJECTED TO DIAMETRAL COMPRESSION.

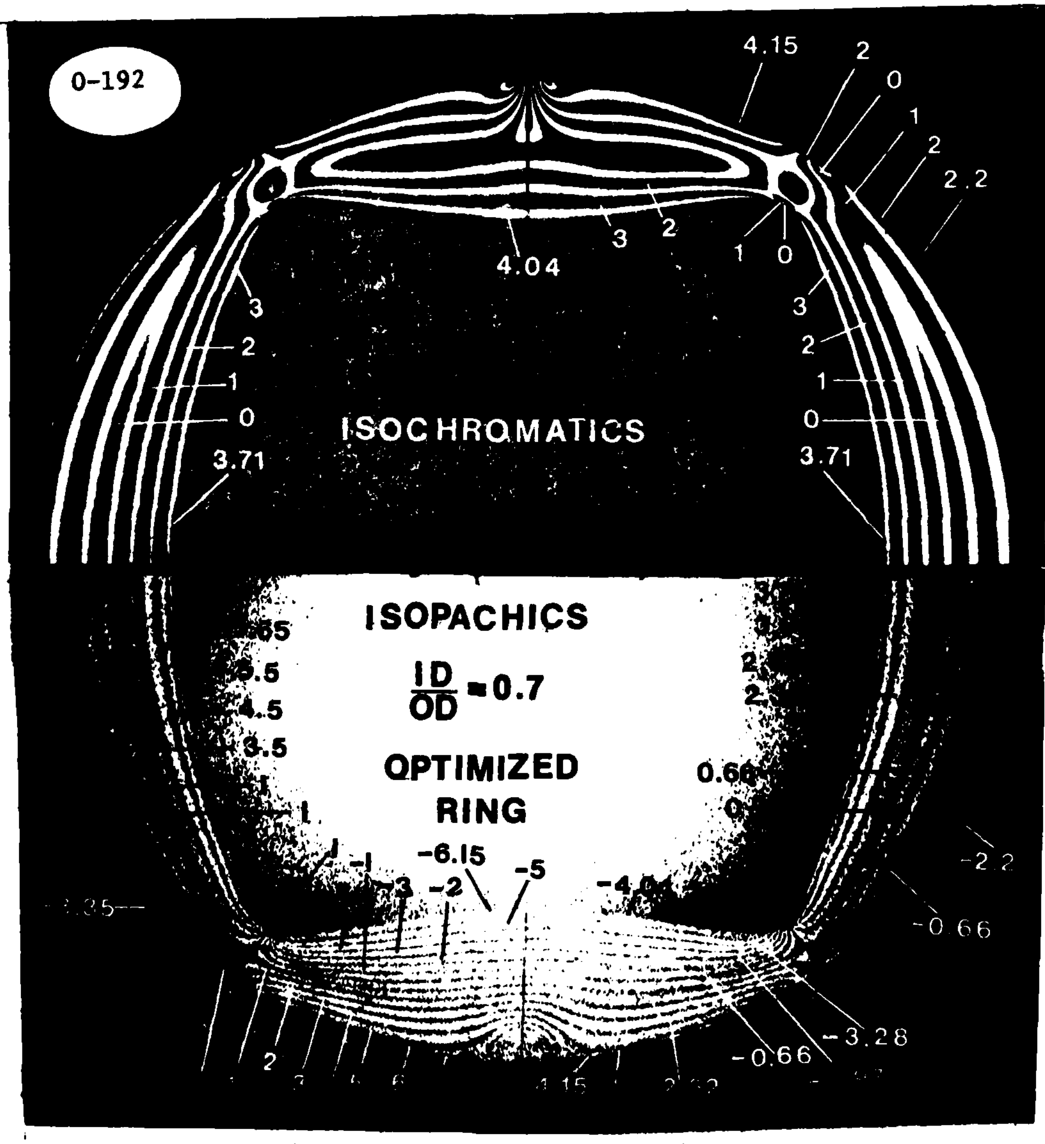
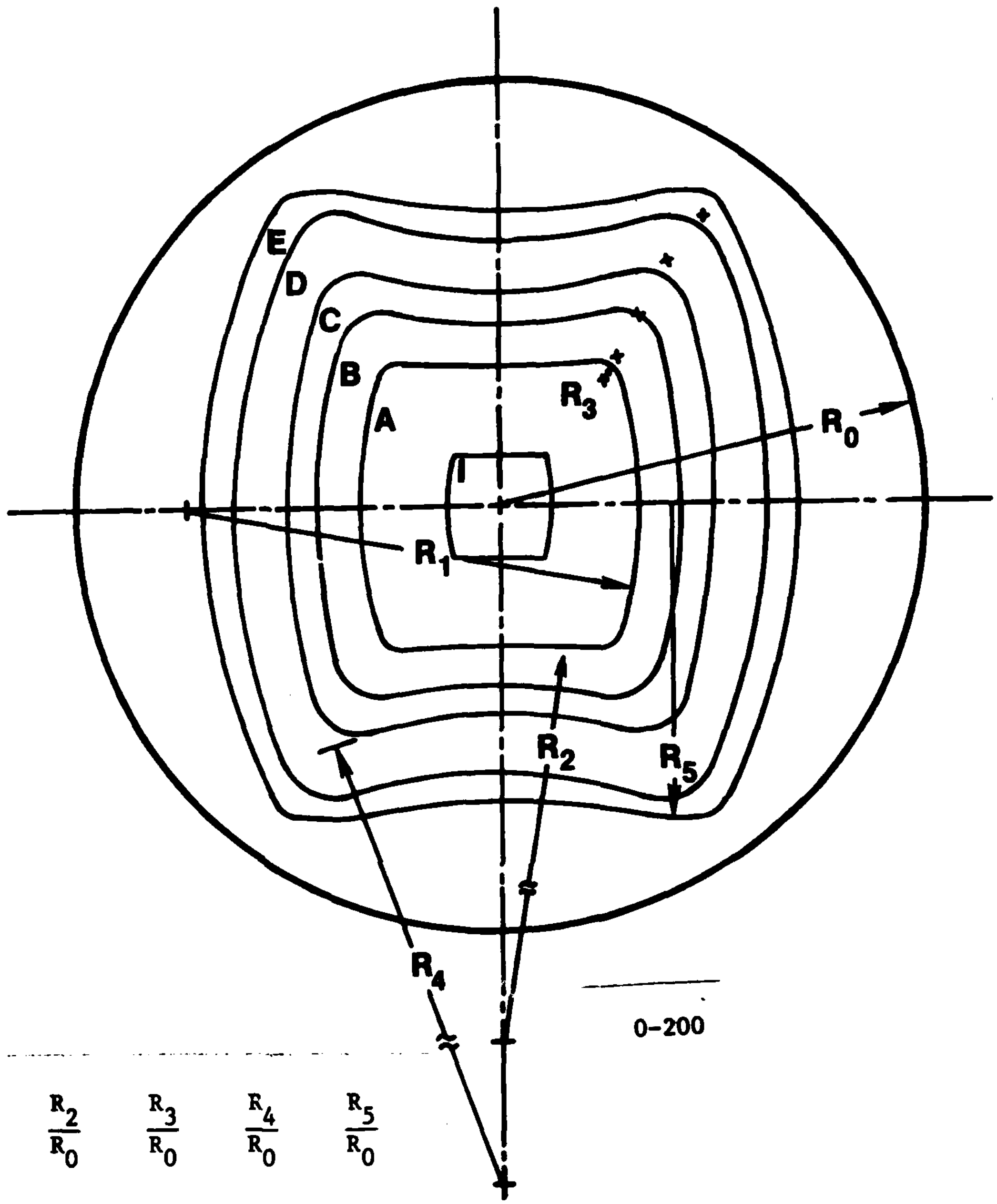


FIG. 18 DARK FIELD ISOCHROMATICS FROM PHOTOELASTICITY AND ISOPACHICS FROM HOLOGRAPHIC INTERFEROMETRY FOR THE OPTIMIZED RING (ID/OD = 0.7). (ISOPACHICS ORDERS ON RIGHT SIDE, NORMALIZED).



	$\frac{ID}{OD}$	$\frac{R_1}{R_0}$	$\frac{R_2}{R_0}$	$\frac{R_3}{R_0}$	$\frac{R_4}{R_0}$	$\frac{R_5}{R_0}$	
A	0.33	1.06	1.71	0.05	-	-	
B	0.43	1.43	1.44	0.11	-	-	
C	0.53	1.73	1.01	0.10	1.25	-	
D	0.63	1.51	1.13	0.12	-	-	
E	0.70	1.49	1.49	0.06	-	0.73	
I	0.14	0.35	∞	0.05	-	-	(Estimated)

FIG. 19 OPTIMUM SHAPE OF THE INNER BOUNDARY OF CIRCULAR RINGS SUBJECTED TO DIAMETRAL COMPRESSION.

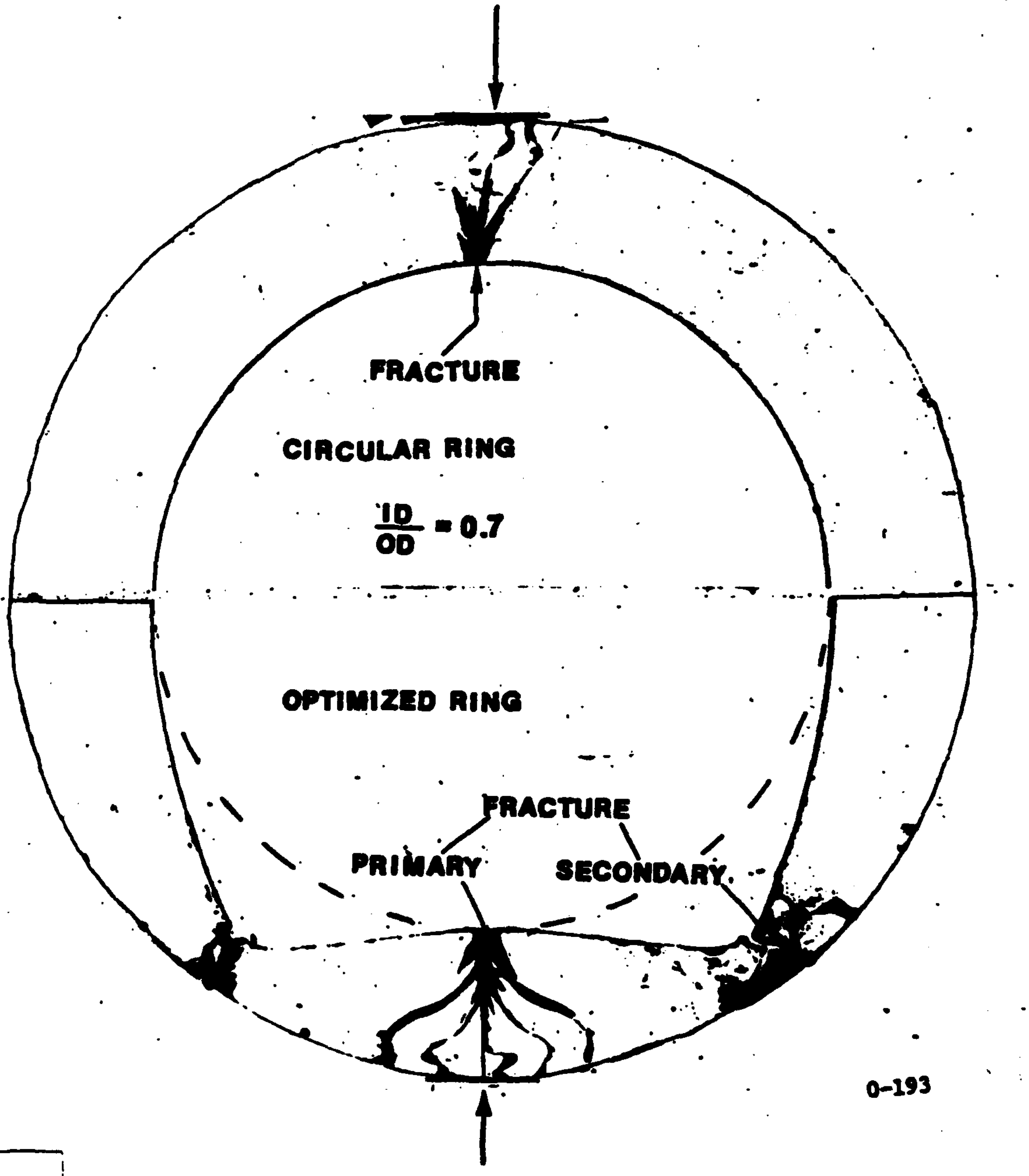


FIG. 20 FRACTURE MECHANISMS IN THE CIRCULAR AND OPTIMIZED RINGS

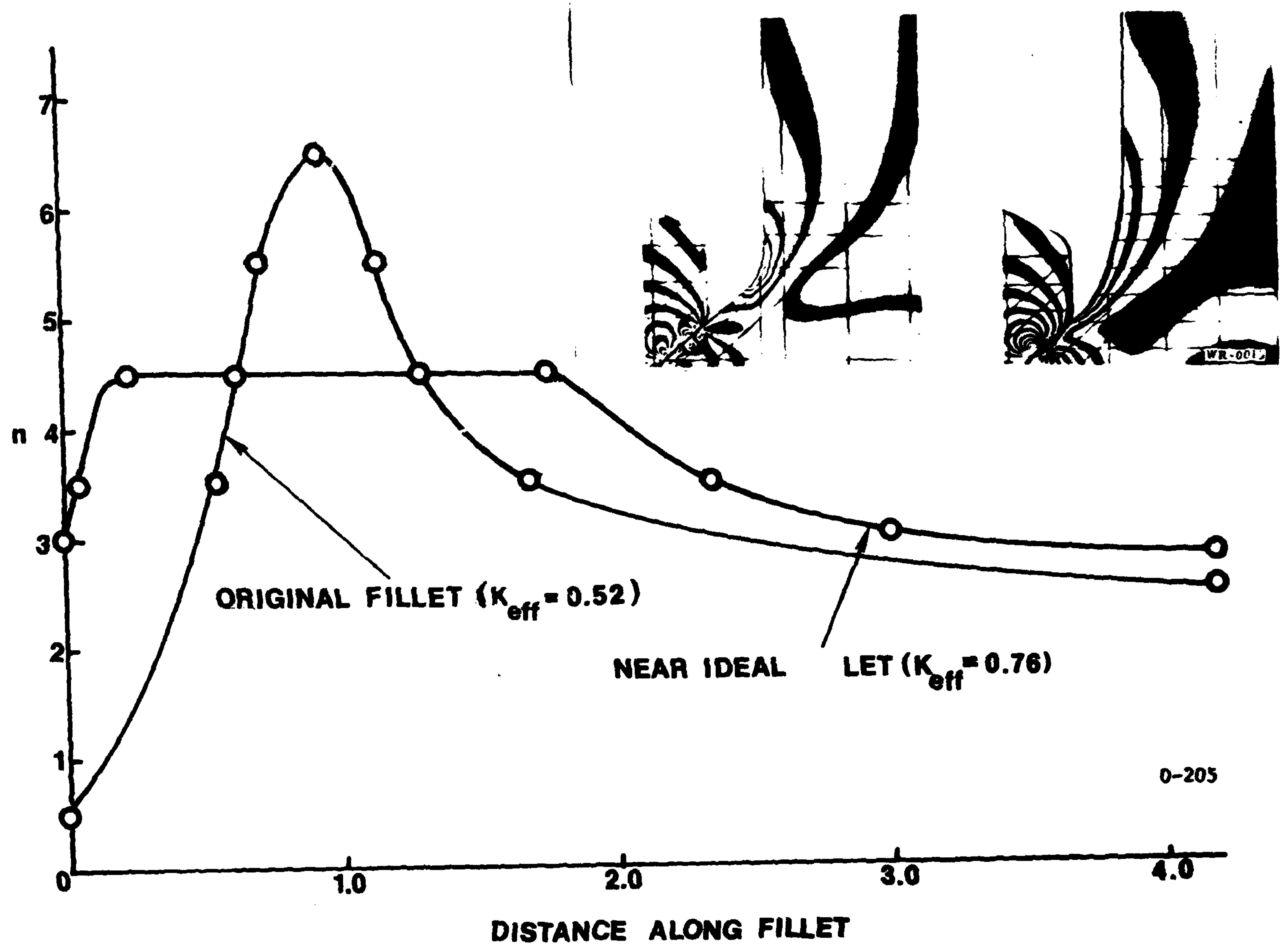
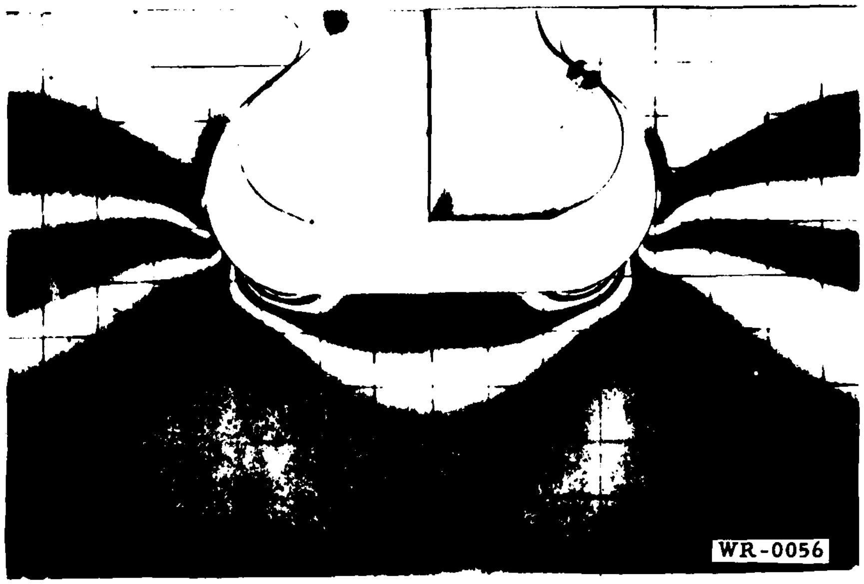
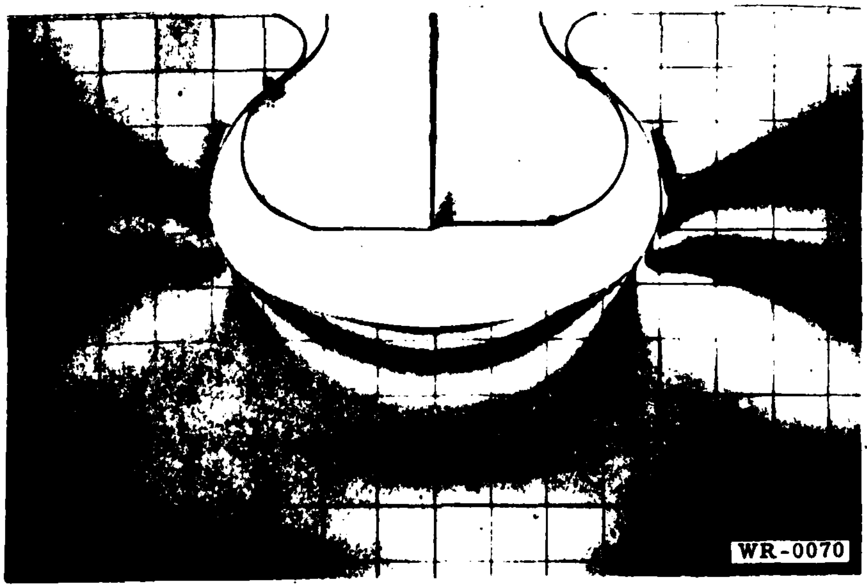


FIG. 21 FILLET AT THE DOVE TAIL JOINT BETWEEN BLADES AND ROTOR.



ORIGINAL DESIGN



REDESIGN

0-210

FIG. 22 PHOTOELASTIC ISOCHROMATIC PATTERNS PRODUCED BY A TANGENTIAL LOADING IN THE ORIGINAL AND REDESIGNED MODELS OF THE SLOT OF A TURBINE ROTOR.

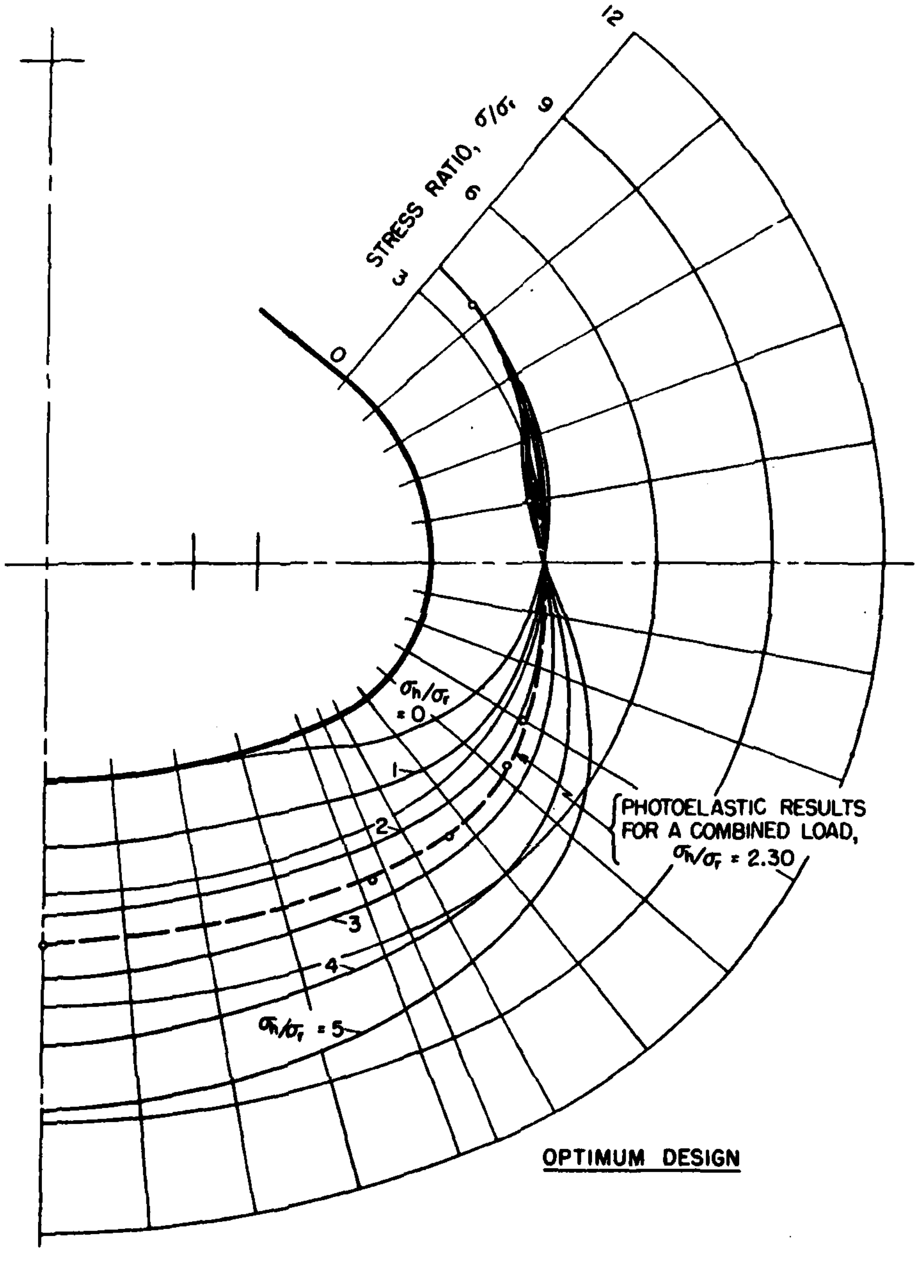


FIG. 23 BOUNDARY STRESS DISTRIBUTIONS FOR VARIOUS RATIOS OF TANGENTIAL LOADING TO RADIAL BLADE LOADING IN THE REDESIGNED MODEL OF THE SLOT OF A TURBINE ROTOR.

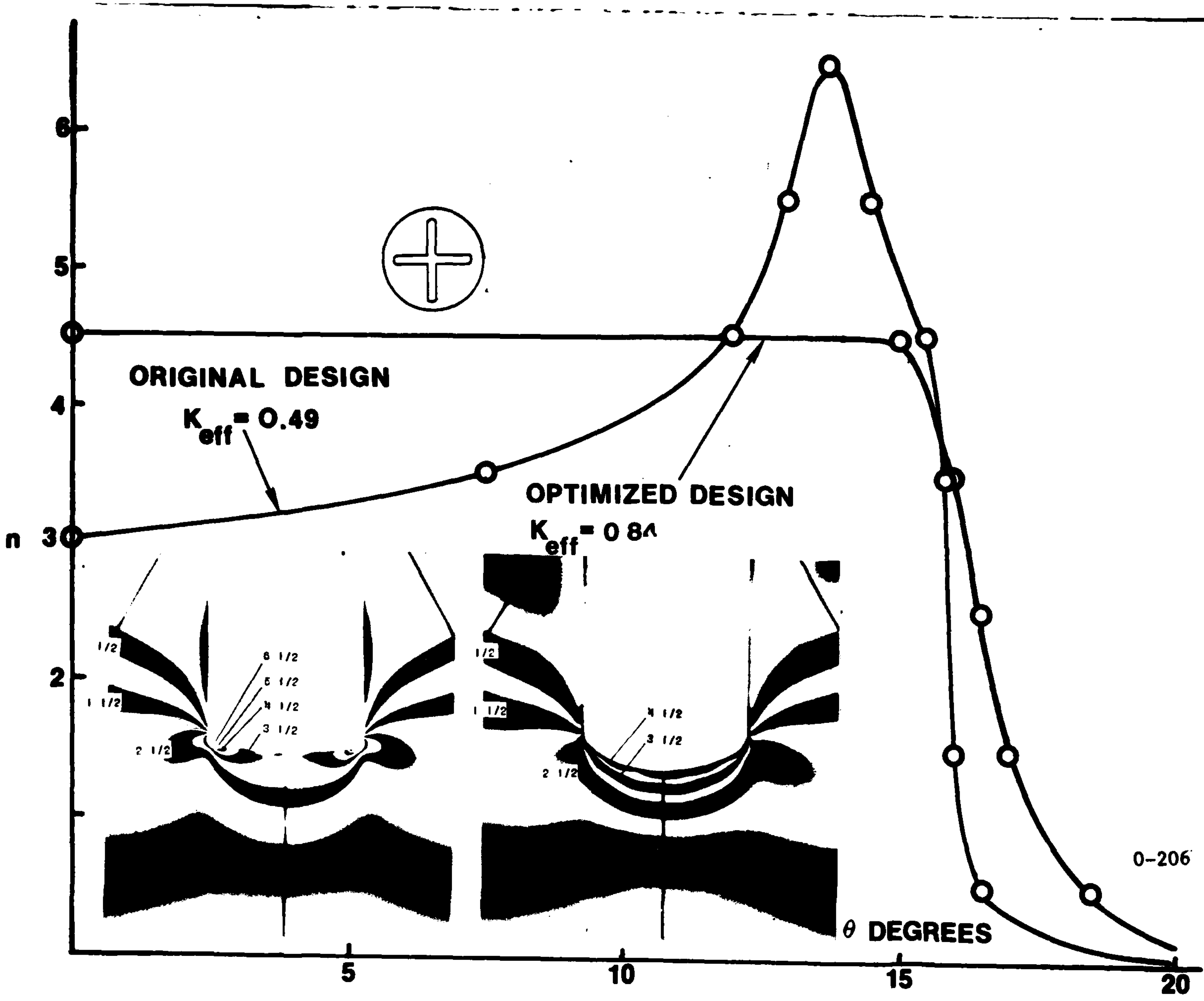


FIG. 24 BOTTOM OF THE RAYS OF STAR-SHAPED CONFIGURATION OF PROPELLANT GRAIN WITH STRAIGHT SIDED RAYS.

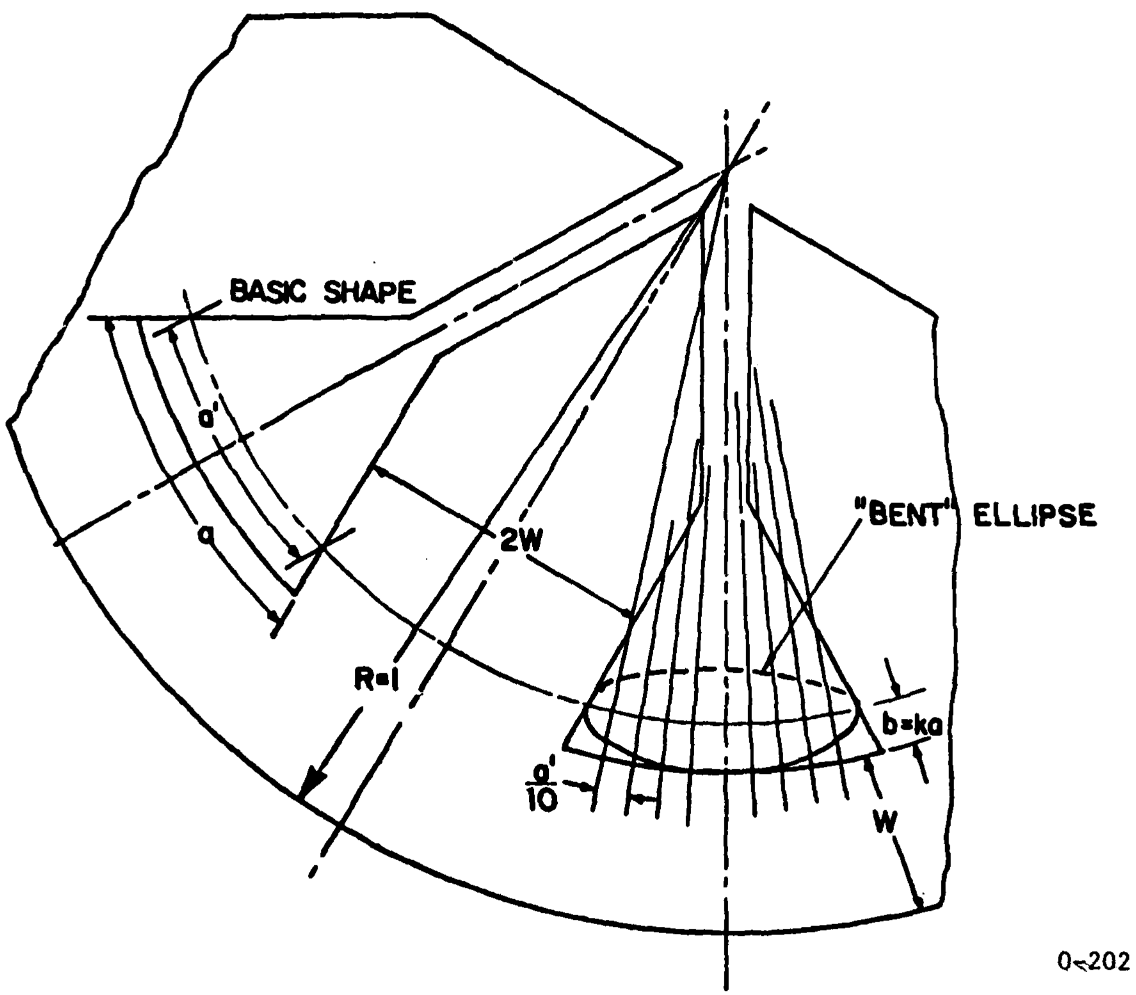
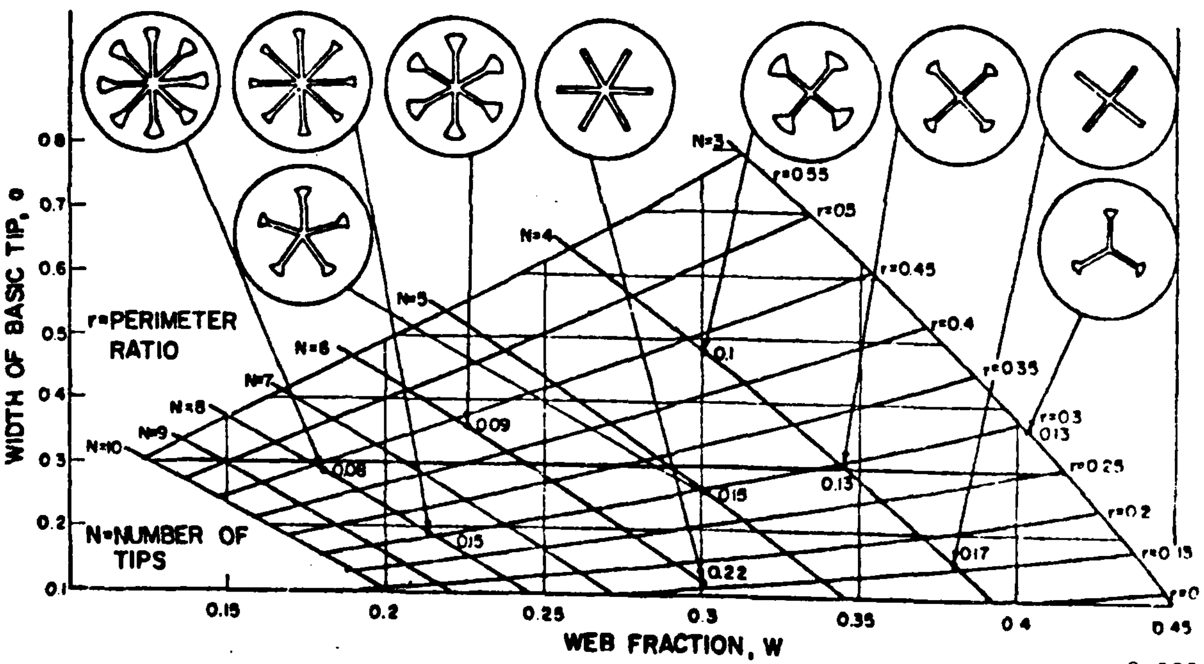


FIG. 25 GEOMETRY OF ENLARGED TIP OF RAYS IN STAR PERFORATED DISKS AND METHOD OF LAYING OUT THE TIP FORM.

THE VALUES GIVEN CORRESPOND TO OPTIMUM b/a FOR THE PARTICULAR GEOMETRY



0-203

FIG. 26 RELATIONSHIP BETWEEN THE GEOMETRIC PARAMETERS IN STAR PERFORATED DISKS WITH ENLARGED TIP OF RAYS.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 53	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) LIGHTER AND STRONGER		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) A. J. Durelli and K. Rajaiah		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Oakland University Rochester, MI 48063		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Department of the Navy Washington, D.C. 20025		12. REPORT DATE February 1980
		13. NUMBER OF PAGES 52
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of this report is unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Optimization
Photoelasticity
Weight reduction
Stress concentrations
Propellant grains
Turbine dove-tails

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A new method has been developed that permits the direct design of shapes of two-dimensional structures and structural components, loaded in their plane, within specified design constrains and exhibiting optimum distribution of stresses. The method uses photoelasticity and requires a large field diffused light polariscope. Several problems of optimization related to the presence of holes in finite and infinite plates, subjected to uniaxial and biaxial loadings, are solved parametrically.

Some unexpected results have been found: (1) the optimum shape of a large hole in a bar of finite width, subjected to uniaxial load, is "quasi" square, but the transverse boundary has the configuration of a "hat"; (2) for the small hole in the large plate, a "barrel" shape has a lower s.c.f. than the circular hole and appreciably higher coefficient of efficiency; (3) the optimum shape of a tube, subjected to diametral compression, has small "hinges" and is much lighter and stronger than the circular tube. Applications are also shown to the design of dove-tails and slots in turbine blades and rotors, and to the design of star-shaped solid propellant grains for rockets.